THE

ANALYST:

OR, AN

INTRODUCTION

TO THE

MATHEMATICS.

THE ANALYST:

OR, AN INTRODUCTION TO THE MATHEMATICS.

CONTAINING,

I. THE Doctrine of *Vulgar* and *Decimal Fractions*, wherein the Fundamental Principles are fully and clearly explained in all their Cases.

II. THE Extraction of *Roots*, according to the *Newtonian* Method, much preferable to that now taught in Schools.

III. THE First Principles of *Algebra* demonstrated in a very short and easy Method, illustrated with variety of Instances, in the Application thereof to the Solution of Problems.

For the Use of Schools as well as of Private Gentlemen.

Θήγει φρένα τὸ ἡδὺ τεχνῶν.

LONDON:

Printed by E. CAVE, for J. FULLER, at the *Bible* and *Dove*, *Ave-mary-lane*.

M.DCC.XLVI.

THE

DOCTRINE

OF

VULGAR FRACTIONS.

DEFINITION I.

UNITY is an abstract Idea of all those Things we call *one*, and is therefore put for 1, and these are either the same or different. I say, Unity, Unit, or One is, by which we distinguish, discern, know, name or express any Thing that is, to be one.

2. Number is a Multitude, or a Many, of Units.

ILLUSTRATION.

As Unit, or One, is that, by which every Thing that is, is expressed to be one; so Number is that by which we express what contains Quantity or Multitude of those Things, Quantities, or Magnitudes we desire to have named, known or signified: As two Men, three Books, six Things, nine Months, forty Pounds of Money, sixty hundred Weight, two thousand Ells, and so *in infinitum*. If the Number be referred to, or compared with a given Unity, it is called a determined Number; but if referred to an indefinite Unity, it's an undetermined Number, or Quantity; as for the *Length* of

a Bowling-green, if that is to be determined, we must first assume some Quantity at Pleasure for Unity, and find the Relation it hath thereto; thus the *Length* will be expressed differently, according to the different Unity assumed.

3. Part is one Number of a greater of the same Kind, when the lesser measures the greater Number.

ILLUSTRATION.

One Number is said to measure the greater of the same Kind, when the lesser is many times contained in the greater exactly; as 2 doth measure 6, for 2 is contained in 6, 3 times exactly; 4 doth measure 32, for 4 is contained in 32, 8 times, therefore 4 is the one eighth Part of 32; and so on *in infinitum.* Hence it is manifestly evident, that Unit measures all Numbers, and is a Part of any Number; as of 8, it is the one eighth, of 20 it is the one twentieth Part, *&c.*

4. Parts is one Number of a greater of the same Kind, when the lesser Number doth not measure the greater.

ILLUSTRATION.

When the lesser Number doth not measure the greater of the same Kind, then we must conceive that the Unit doth measure the same greater Number, and consequently a Part thereof, and therefore, that the lesser Number is so many such Parts of the greater, as the same lesser Number doth contain Units. As 7 doth not measure 15, but the Unit doth measure them both, for of the former it is one seventh, and of the latter it is the one fifteenth; likewise 9 doth not measure 47, but Unit measures them both, for of the former Part it is the one ninth, and of the latter one forty seventh Part, and therefore 7 is seven fifteenth Parts of 15, and 9 is nine forty seventh Parts of 47, and so *in infinitum.*

5. Many times is one Number of a lesser of the same Kind, when the lesser Number doth measure the greater.

ILLUSTRA-

ILLUSTRATION.

This is the Converse of the third Definition; for as the lesser Number is such a Part of a greater Number of the same Kind, so the greater Number is so many times of the lesser Number of the same Kind; as 9 is measured by 3 at 3 times, consequently 9 is 3 times of 3.

6. Things may be said to be equal, when one may be substituted in the Room of the other, without changing the Quantity; and tho' two Things be unequal, a Part of the greater may be substituted for the Whole of the other, or the lesser may be substituted for the Part of the other, without changing the Quantity.

7. An *aliquot* Part is that which being repeated a certain Number of Times becomes equal to the Whole, but when it becomes either more or less, it is an *aliquant* Part; therefore a lesser Number is an *aliquot* Part of a greater, when the lesser Number measures the greater Number exactly, and, by *Definition the 2d of the 5th Book of* Euclid, the greater is then called a *Multiple* of the lesser, that is, a Magnitude of a Magnitude, the Greater of the Lesser. Thus 6 is an *aliquot* Part of 30, and therefore by this Definition 30 is a Multiple of 6, for 6 times 5 is 30; but 8 is an *aliquant* Part of 30, because 8 does not measure 30, for 3 times 8 is 24, and therefore deficient of 30. Generally one Number is said to be *multiple* to another, when it contains it a certain Number of Times without a Remainder, and the Number contained is the *submultiple*.

8. Quantities are *commensurable* when there is an *aliquot* Part common to both, or when one is an *aliquot* Part of the other, otherwise they are called *incommensurable*.

9. Quantities are *homogeneous*, when one of them taken a certain Number of Times will exceed the other, or subtracted will leave nothing, or a Number less than itself; but when one of them repeated cannot exceed the other, they are *heterogeneous*.

10. Numbers are either abstract, as 4, 5, 6, or con-

crete, when we determine what they are; and if they are referred to the ſame Unity, they are called *homogeneous* Numbers, but if to different, they are *heterogeneous*; as 5 Globes of Gold, and 10 of Silver. Therefore a whole Number may be abſtract, that is, having no Denomination to it, or concrete; but a Fraction may wholly be concrete, that is, having a Denomination annex'd to it, or partly abſtract, or partly concrete. For *three fifths*, conſidered as a Fraction, is *abſtract*, ſince it is not expreſſed what it is the *three fifths* of, but in reſpect of its Denomination *fifths*, it is *concrete*; alſo three fifth Parts of a Yard is *concrete*, both in reſpect of its Denomination *fifths*, and in reſpect of what they are fifths of.

11. A prime Number is that which can only be meaſured by Unity, as 5, 7, 11.

12. A compoſite Number is that which other Numbers beſides Unity will meaſure, as 6 meaſures 12 by 2, and 2 meaſures 12 by 6.

13. A common Meaſure is a Number by which two or more Numbers are meaſured; as 6 meaſures 12 and 30, being twice in the one, and 5 times in the other.

14. Numbers that are prime to one another, are ſuch as have no other common Meaſure but an Unit.

15. An Integer or whole Number is referred to Unity, as the Whole to a Part, but a broken Number as a Part to the Whole, and is expreſſed by 2 Numbers, called a Vulgar Fraction, one above, which is the Numerator, and the lower the Denominator; for the Whole is broken into a certain Number of Parts, and the upper Number is ſo many of thoſe Parts; as $\frac{4}{5}$, *four fifths*. Therefore it ſhews that the Integer is divided into 5 Parts, and the Line above exhibits that 4 of thoſe 5 Parts are taken or expreſſed, and therefore 4 is called the Numerator, and 5 its Denominator, becauſe it denotes the Name, or Denomination thereof, and generally $\frac{N}{D} = \frac{\text{Numerator}}{\text{Denominator}}$.

16. A rational Number is that which can be meaſured by

by Unity; an irrational cannot, and ſo is called a Surd, or inexpreſſible.

17. A whole Number is that which numbereth Wholes.

18. A broken Number, or Fraction, numbereth the Parts of an Unit.

19. A mixt Number is made by putting a Fraction to a whole Number.

20. A perfect Number is that which is equal to all its own aliquot Parts taken together.

21. A diminutive Number is greater than the Sum of all its aliquot Parts.

22. An abounding Number is leſs than the Sum of all its aliquot Parts.

23. An abſtract Number is that which hath no Denomination annexed to it.

24. A concrete Number is ſuch as hath a Denomination annexed to it.

25. A ſingle Fraction is ſuch as denotes a Part or Parts of ſome Whole.

26. A compound Fraction is ſuch as denotes a Part or Parts of ſome Part or Parts.

27. A proper Fraction is leſs than an Unit.

28. An improper Fraction is greater than an Unit.

29. Ratio is, that by which is conſidered, known, named and expreſſed what one Quantity is, of another of the ſame Kind; as that it is greater, leſſer or equal. But, more preciſely, Ratio is the Conſideration and Knowledge of the Quantity or Number, by which it is exactly expreſſed and named, what Quantity one Quantity is, of another of the ſame Kind; therefore Ratio is the Relation of Things homogeneous, which determines the Quantity of one from the Quantity of the other, without uſing another homogeneous; the Things compared are called the Terms of the Ratio, that which is referred to the other being the Antecedent, and that to which it is referred, the Conſequent.

COROLLARY I.

Seeing in Fractions the Relation of the Numerator to

the Denominator is underſtood without aſſuming a third homogeneous, the ſame will expreſs a Ratio.

COROLLARY II.

If two Quantities are compared together without aſſuming a third, either they are equal or not, that is, it is either a Ratio of Equality or Inequality.

COROLLARY III.

If the Terms of the Ratio be unequal, either the leſſer is referred to the greater, or the greater to the leſſer; the former as a Part to the Whole, the latter as the Whole to the Part. The Ratio will thus determine how often the leſſer is contained in the greater, or how often the greater contains the leſſer, that is, how much of the greater the leſſer is equal to, which Diviſion diſcovers.

COROLLARY IV.

A Ratio being in inſelf intelligible, we may thereby diſcern the Relation of Things, tho' they are not preſent to Senſe, or compared with one another.

30. A Ratio is ſaid to be rational, which is as 1, or as a rational Number to another, as 2 to 3; but it's irrational, when it cannot be expreſſed by rational Numbers. *Example.* Let there be two Quantities, as A and B, and let A be leſs than B; if you take A from B as often as it can be taken, ſuppoſe 7 times, there will remain either ſomething or nothing; in the latter Caſe A will be to B as 1 to 7, or A will be equal to $\frac{1}{7}$ B: Thus it is rational. In the former Caſe, either there will be a Part, which being taken, ſuppoſe 4 times from A, and 9 times from B, leaves nothing, or there will be no ſuch Part of the former, then A will be to B as 4 to 9, or A equal $\frac{4}{9}$ B, and ſo the Ratio is rational; but if the latter, the Ratio of A to B cannot be expreſſed in rational Numbers; that is, it cannot be told what Part A is to B.

31. The Exponent of the Ratio is the Quotient that ariſeth

ariseth from dividing the Antecedent by the Consequent. *Example.* Exponent of the Ratio of 3 to 2 is $1\frac{1}{2}$, but of 2 to 3 is $\frac{2}{3}$,

COROLLARY I.

If the Consequent be 1, the Antecedent is the Exponent of the Ratio, as the Exponent of the Ratio of 2 to 1 is 2, of 4 to 1 is 4, Thus an Integer or whole Number expresses the Ratio of many to one, or of a Multitude to Unity.

COROLLARY II.

The Exponent of the Ratio is to Unity, as the Antecedent to the Consequent.

COROLLARY III.

Ratios are understood by their Exponents, and therefore if the Antecedent be A, and the Consequent B, the Ratio of A to B may be expressed thus, $\frac{A}{B}$.

32. If the lesser Term be an *aliquot* Part of the greater, the Ratio of greater Inequality is called *multiple*, but the Ratio of lesser Inequality *sub-multiple*; particularly, if in the first Case the Exponent be 2, the Ratio is double, if 3, triple, &c. In the other Case, if the Exponent be $\frac{1}{2}$, the Ratio is sub-double, if $\frac{1}{3}$ subtriple; as 2 to 6 is a sub-triple Ratio, but 6 to 2 is a triple Ratio.

COROLLARY.

If A be to B as C to D, then the Identity of the Ratio may be expressed thus, $\frac{A}{B} = \frac{C}{D}$, or thus, A : B : : C : D.

33. The Similitude or Identity of two Ratios is called Proportion, and the Quantities that have the same Ratio are said to be proportional. *Example.* If A be to B as C to D, then these 4 Numbers are proportional.

34. Proportion is Similitude, or Likeness of *Ratios*, and doth consist at the least in three Terms.

ILLUSTRA-

Illustration.

Proportion is Similitude, or Likeneſs, of *Ratios*, between the Quantities compared; as of 4 to 2, and 10 to 5, the *Ratio* is 2 times, or double, in both Compariſons. Therefore Proportion doth conſiſt at the leaſt in three Terms, and that is, when the Conſequent is taken as an Antecedent to the next Conſequent, as 18 to 6, and 6 to 2; the *Ratio* is 3 times, or triple in both Compariſons, and ſo *in infinitum*.

THEOREM I.

IF a Number cannot be divided by ſome prime Number, nor greater than the Square Root thereof, that Number is a Prime.

THEOREM II.

IF two Fractions are equal to one another, the Numerator of the one is in ſuch Proportion to its Denominator, as the Numerator of the other to its Denominator.

THEOREM III.

IF the Numerator of a Fraction be leſs than the Denominator, that Fraction is a proper one. But if the Numerator be equal or greater than the Denominator, the Fraction is improper.

THEOREM IV.

EVERY whole Number may be expreſſed in the Form of a Vulgar Fraction, without altering its Value, if that whole Number be the Numerator, and Unity the Denominator.

THEOREM V.

QUantities that have the ſame Ratio that one rational Number hath to another, are commenſurable.

DEMON-

DEMONSTRATION.

An Unit is an aliquot Part of a rational Integer, and a Fraction hath an aliquot Part in common with an Unit; therefore ſuch as are to one another, as one rational Number to another, one of them is either an aliquot Part of the other, or elſe there is one aliquot Part common to both, therefore they are commenſurable.

THEOREM VI.

COmmenſurable Quantities among themſelves are either as an Unit to a rational whole Number, or as a rational whole Number is to another; but it is not ſo with Incommenſurables.

DEMONSTRATION.

In commenſurable Quantities, either one is an aliquot Part of the other, or elſe there is an aliquot Part common to both; and if in the former Caſe, the leſſer Quantity be taken for Unity, and in the latter a common aliquot Part, in the firſt Caſe it will anſwer the greater Quantity, and in the ſecond Caſe the rational whole Number will anſwer both; therefore, in the firſt Caſe the Quantities are among themſelves as Unity; and in the ſecond as a rational whole Number to another, which was the firſt Part.

In Incommenſurables there is no common aliquot Part (*as by Definition* 8.) therefore there is no Unity to which they can be commenſurable; wherefore ſeeing every rational Number is commenſurable with Unity, they connot be as one rational Number to another, which was the ſecond Part.

THEOREM VII.

TWO Quantities multiplying each other, have the ſame Product.

DEMONSTRATION.

Let the two Factors be A and B, then 1 to A, B will be

be to A B; and 1, to B, A will be to B A; therefore AB is equal B A. *Example*, $7\times5=5\times7=35$.

COROLLARY.

Let there be three Factors A, B and C; becauſe A B=B A, then C A B=C B A, and ſo ABC=BAC; and becauſe B C=C B, then A C B=A B C, and ſo C B A=B C A; wherefore C A B=C B A=A B C= B A C=A C B=B C A, that is, there is the ſame Product in whatever Order they are multiplied.

THEOREM VIII.

ALL Products are compoſite Numbers.

This is ſelf-evident, and needs no Demonſtration.

THEOREM IX.

IF the Numerator be equal to the Denominator, as $\frac{5}{5}$ the Fraction is equivalent to an Integer; if leſs, it is leſs than an Integer; if more, it is greater.

DEMONSTRATION.

For the Denominator ſhews the Whole broken into ſo many Parts, and the Numerator is ſo many of thoſe Parts (*by Definition* 14.); if then they be equal, the Fraction is equal to the very Integer itſelf; as $\frac{3}{3}$ of a Groat is a Groat itſelf; if leſs, then there are ſome Parts taken, but not all, and ſo it is leſs; if more, then there are more Parts taken than the Integer hath, and ſo the Integer is equal to a Part of the Fraction, and ſo greater.

SCHOLIUM.

Thoſe that are equal or greater than the Integer are ſpurious, or improper, Fractions.

THEOREM X.

IF the Conſequents of two Ratios be equal, they are as the Antecedents; but if the Antecedents be equal, they are reciprocally as the Conſequents.

DEMON-

DEMONSTRATION.

For $\frac{7}{1} : \frac{9}{1} : 7 : 9$
And $\frac{1}{9} : \frac{1}{7} :: 7 : 9$.

Characters used in the following Sheets, for the more convenient operating the Rules, are as follows.

$+$ IS a Sign, denoting Addition; as if $5+7+9$, &c. then it would be the same as 5 added to 7, and that Sum added to 9, &c.

$-$ Is a Sign, denoting Subtraction; as $7-5$, shews that you must subtract 5 from 7.

$\times$ Is a Sign, denoting Multiplication; as $5\times7\times9$, &c. shews that 5 is to be multiplied by 7, and that Product again by 9, &c.

$\div$ Is a Sign, denoting Division, as $8\div4$ shews that 8 is to be divided by 4. But in the following Sheets I more particulary use Division thus $\frac{8}{4}$, [the top Figure always denoting the Dividend, and the bottom the Divisor; and it evidently follows that when the top Number is less than the bottom, then it is self-evident that it must stand so, and from thence are derived all Vulgar Fractions. *Example.* If $\frac{8}{4}$, then the Quotient will be 2, which I set thus, $\frac{8}{4}=2$; also $\frac{56}{4}=14$. But if $\frac{11}{17}$, then it cannot be divided by 17, therefore it must so stand, and consequently a Vulgar Fraction. I also denote Division thus 4)8(2, and 4)56(=14; and so the rest.

$=$ A Sign, denoting Equality, as $4+2=6$, that is, 4 and 2 is equal to 6.

: : A proportional Sign, as if 2 Yards coſt 5 Pounds, 4 Yards will coſt 10 Pounds, which I ſet thus, 2 : 5 : : 4 : 10.

$\surd$ Is a Sign denoting the Square-Root.

$\sqrt[3]{\ }$ or $\overline{\qquad}|^{\frac{1}{3}}$ Is a Sign, denoting the Cube-Root, and $\sqrt[4]{\ }$ or $\overline{\qquad}|^{\frac{1}{4}}$ the Biquadratic Root.

Having treated of the Definitions, Theorems, with their Demonſtrations, *&c.* being the Foundation for ſuch a neceſſary Work, it is now time we ſhould come to explain the Subject itſelf, as it is a very juſt Obſervation, that Arts are more eaſily attained by Examples than Precepts.

OF

OF

VULGAR FRACTIONS.

ANY whole Thing, *or an* Unit, *may be considered as* divided *into any Number of* equal Parts, *which have their* Name *from the* Number *contained in that* Unit.

As if Unit be conceived to be divided into *five* equal *Parts*, thoſe *Parts* are called *fifths*, and are thus written $\frac{}{5}$.

And any *Number* of thoſe *Parts*, ſuppoſe *three*, are thus expreſſed, *three fifths*, or three divided by five, and thus written $\frac{3}{5}$, and called a Fraction.

COROLLARY I.

Whence it follows, that a *Fraction* is but a *Quotient*, ſignifying a *Part* or *Parts* of an Unit, expreſſed by a *Numerator* as a *Dividend*, and a *Denominator* as a *Diviſor* (*See Definition* 15, *alſo page* 11.)

Thus *one ſeventh* Part of any *Thing* is the *Quotient* of that *Thing divided* by *ſeven*, which by *common Diviſion* is expreſſed thus, $\frac{1}{7}$.

Alſo $\frac{3}{4}$ ſignifies *three fourth* Parts of an *Unit*, or *one fourth* Part of *three Units*.

COROLLARY II.

As the Numerator is to the Denominator,
So is the Fraction to Unit.
For the Dividend is to the Diviſor,
As the Quotient is to Unit.
Thus $4 : 5 : \frac{4}{5} : 1$.

C COROL-

COROLLARY III.

Fractions having the same *Denominators* are one to another as their *Numerators*.

Thus $\frac{3}{7} : \frac{4}{7} :: 3 : 4$

For $3 : \frac{3}{7} :: 7 : 1$ } by preced. Cor.

And $4 : \frac{4}{7} :: 7 : 1$ }

Theref. $\frac{3}{7} : \frac{4}{7} :: 3 : 4$.

COROLLARY IV.

As any Fraction is to Unit,
So is Unit to the reverse Fraction.

Thus $\frac{4}{7} : 1 :: 1 : \frac{7}{4}$

For $\frac{4}{7} : 1 :: 4 : 7$ } by Cor. 2.

And $1 : \frac{7}{4} :: 4 : 7$ }

Theref. $\frac{4}{7} : 1 :: 1 : \frac{7}{4}$.

SCHOLIUM.

Since it often happens, in *reducing*, *adding*, *subtracting*, &c. *Fractions*, that they swell into too great Numbers, which are more troublesome, and not so manageable as smaller ones, therefore we shall in the next Place shew the way of *reducing* them into their *least Terms*, either before or after such Operations, as there is occasion; which is done by the help of the following

PROBLEM.

TWO Numbers being given, to find their greatest common Measure, (i. e.) *the greatest Number that can* divide *both without* remainder.

RULE.

Divide the greatest by the least, and that Divisor by the Remainder continually, till nothing remain, and the last Divisor will be the greatest common Measure.

EXAMPLE I.

If the Numbers be 91 and 117, or $\frac{91}{117}$. The greatest common Measure is 13. For

OPERATION.

```
91)117(1
   91
   26)91(3
      78
      13)26(2
         26
```

Or thus

```
117|1
 91|3
 26|2
 13|
```

EXAMPLE II.

If the Numbers be 468 and 846, or $\frac{468}{846}$. The greatest common Measure is 18; for

OPERATION.

```
468)846(1
    468
    378)468(1
        378
         90)378(4
            360
             18)90(5
                90
                ..
```

Or thus

```
846|1
468|1
378|4
 90|5
 18|
```

EXAMPLE III.

If the Numbers be 5808 and 10080, or $\frac{5808}{10080}$. The greatest common Measure is 48.

OPERATION.

```
5808)10080(1
     5808
     ----
     4272)5808(1
          4272
          ----
          1536)4272(2
               3072
               ----
               1200)1536(1
                    1200
                    ----
                     336)1200(3
                         1008
                         ----
                          192)336(1
                              192
                              ---
                              144)192(1
                                  144
                                  ---
                                   48)144(3
                                      144
                                      ---
                                       ..
```

Or thus

10080	1
5808	1
4272	2
1536	1
1200	3
336	1
192	1
144	3
48	

EXAMPLE IV.

What is the greatest common Measure of 46198 and 871624?

OPERATION.

```
46198)871624(18
      46198
      409644
      369584
       40060)46198(1
             40060
              6138)40060(6
                   36828
                    3232)6138(1
                         3232
                         2906)3232(1
                              2906
                               326)2906(8
                                   2608
                                    298)326(1
                                        298
                  28)298(10             28
                     280
                      18)28(1
                         18
                         10)18(1
                            10
                             8)10(1
                               8
                               2)8(4
                                 8
                                 ..
```

Or thus

871624	18
46198	1
40060	6
6138	1
3232	1
2906	8
326	1
298	10
28	1
18	1
10	1
8	4
2	

Note. *If* 1 *be the* greateſt common Meaſure, *the Numbers are ſaid to be* Prime *to one another.*

PROPOSITION I.

GIVEN two or more Numbers, to find a *common Multiple* to them.

RULE.

Multiply continually all the given Numbers, the Product is the Number sought.

EXAMPLE.

Find a common Multiple to 2, 4, 5, 6 and 8.
Answer. Thus $2\times4=8\times5=40\times6=240\times8=1920$.

PROPOSITION II.

GIVEN two Numbers, to find their least *common Multiple.*

RULE.

Find first their greatest common Measure, then divide by that Measure either of the Numbers, and the Quotient multiplied by the other of those Numbers, the Product will be the Number sought.

EXAMPLE.

What is the least common Multiple of 16 and 24.

Thus 16)24(1
16
greatest C. M. =8)16(2. 8)24(3×16=48. *the Ans.*

PROPOSITION III.

TO reduce a Fraction *into its* least Terms.

RULE.

Divide the Numerator and Denominator by their greatest common Measure, their Quotients will be a Fraction equivalent to the former, and in the least Terms.

EXAMPLE I.

If the given *Fraction* be $\frac{91}{117}$.
The *greatest common Measure* is 13.
And 13) 91(7 the given Fraction in its least Terms.
13)117(9

Ex-

EXAMPLE II.

If the given *Fraction* be $\frac{468}{846}$.

The *greatest common Measure* is 18.

And $\begin{array}{l} 18)468(26 \\ 18)846(47 \end{array}$ $\frac{26}{47}$ the given Fraction in the least Terms.

Note. *When the greatest* common Measure *is* 1, *the Fraction is already in the smallest Terms.*

COROLLARY.

A *Fraction*, whose Terms are even, may be abbreviated by a *continual Division* by 2.

Thus $\frac{384}{512} = \frac{3}{4}$.

For $\begin{array}{l} 2) \\ 2) \end{array} \left| \frac{384}{512} \right| \frac{192}{256} \left| \frac{96}{128} \right| \frac{48}{64} \left| \frac{24}{32} \right| \frac{12}{16} \left| \frac{6}{8} \right| \frac{3}{4}$

You may also abbreviate a Fraction to its lowest Term, if you will observe this

RULE.

Divide the Numerator and Denominator (if they be both even) by 2, 4, 6, 8, &c. *If the Numerator and Denominator be one even and the other odd, or both odd, try some odd Number, as* 3, 5, 7, 9, &c. *repeat this Division as often as you can, so shall the last Quotient of the Denominator be a new Numerator, and the last Quote of the Denominator, a new Denominator.*

EXAMPLE I.

Reduce $\frac{45360}{75600}$ into its lowest Terms.

OPERATION.

$\overset{12}{\frac{45360}{75600}} \left| \overset{9}{\frac{3780}{6300}} \right| \overset{7}{\frac{420}{700}} \left| \overset{5}{\frac{60}{100}} \right| \overset{4}{\frac{12}{20}} \left| \frac{3}{5} \right.$, so that $\frac{3}{5} = \frac{45360}{75600}$.

EXAMPLE II.

Reduce $\frac{216}{432}$ into its lowest Terms.

OPERATION.

$\overset{4}{\frac{216}{432}} \left| \overset{2}{\frac{54}{108}} \right| \overset{3}{\frac{27}{54}} \left| \overset{9}{\frac{9}{18}} \right| \frac{1}{2}$, so that $\frac{1}{2} = \frac{216}{432}$.

But

But the best and general Way of reducing a Fraction to its lowest Terms, is by the greatest common Measure, which we have exemplified above.

COROLLARY.

When both Terms have Cyphers adjoining cut off equal Cyphers from both.

Thus $\frac{10000}{40000}=\frac{1}{4}$, for $\frac{1|0000}{4|0000}=\frac{1}{4}$.

And $\frac{1000}{1400}=\frac{5}{7}$, for $\frac{10|00}{14|00}=\frac{10}{14}=\frac{5}{7}$.

PROPOSITION IV.

TO reduce an Integer *into an* improper Fraction.

CASE I.

Where there is no Denominator *assigned.*

RULE.

Let the given Integer *be a Numerator, and* Unit *its Denominator.*

Thus $2=\frac{2}{1}$; $11=\frac{11}{1}$, $17=\frac{17}{1}$, $57=\frac{57}{1}$.

For *dividing* by *Unit* does not diminish the *Value*.

CASE. II.

Where there is a Denominator assigned.

RULE.

Multiply the Integer by the assigned Denominator, the Product shall be the Numerator.

EXAMPLE I.

To reduce 13 into an improper Fraction, whose Denominator shall be 12.

Thus $\frac{13\times12}{12}=\frac{156}{12}$, then $\frac{156}{12}$ is the required Fraction.

EXAMPLE II.

Reduce 39 into an improper Fraction, whose Denominator shall be 29.

Thus

Thus $\frac{39\times29}{29}=\frac{1131}{29}$, then $\frac{1131}{29}$ is the required Fraction.

EXAMPLE III.

Reduce 3821 into an improper Fraction, whose Denominator shall be 684.

Thus $\frac{3821\times684}{684}=\frac{2613564}{684}$, then $\frac{2613564}{684}$ is the required Fraction.

For 2613564 : 684 : : 3821 : 1.
And 2613564 : 684 : : $\frac{2613564}{684}$: 1.
Therefore $3821=\frac{2613564}{684}$.

Or $3821=\frac{3821}{1}=\frac{3821\times684}{684\times1}=\frac{3821\times684}{684}=\frac{2613564}{684}$.

PROPOSITION V.

TO reduce mixt Fractions into improper ones.

RULE.

Multiply the Integer by the Denominator of the Fraction, and add the Numerator to the Product, subscribing the same Denominator.

EXAMPLE I.

Reduce $5\frac{3}{4}$ into an inproper Fraction.

Thus $\frac{\overline{5\times4}+3}{4}=\frac{20+3}{4}=\frac{23}{4}$ the required Fraction.

EXAMPLE II.

Reduce $47\frac{3}{5}$, into an improper Fraction.

Thus $\frac{47\times5+3}{5}=\frac{235+3}{5}=\frac{238}{5}$ the required Fraction.

EXAMPLE III.

Reduce $36\frac{3}{4}$ into an improper Fraction.

Thus $36\frac{3}{4}=\frac{36\times4+3}{4}=\frac{144+3}{4}=\frac{147}{4}$.

For

For the *Unit* here is considered as divided into 4 *equal* Parts.

Therefore the *Integers* must be multiplied by 4 to produce 4ths.

To which 3 *fourths* being added, the sum will be also *fourths*.

PROPOSITION VI.

To reduce an *improper Fraction* into an *Integer*, or *mixt Fraction*.

RULE.

Divide the Numerator by the Denominator, and the Quotient will be the Integer, or mixt Fraction required.

Example I.

Reduce $\frac{156}{12}$ into its whole or mixt Number.

Thus $\frac{156}{12}=13$. For 156 divided by $12=13$.

DEMONSTRATION.

For $\frac{156}{12} : 1 :: 156 : 12$.
And $13 : 1 :: 156 : 12$.
Therefore $\frac{156}{12}=13$.

Example II.

Reduce $\frac{147}{4}$ into its whole or mixt Number.

Thus $\frac{147}{4}=36\frac{3}{4}$.

Example III.

Reduce $\frac{25576}{136}$ to its whole or mixt Number.

Thus $\frac{25576}{136}=188\frac{1}{17}$.

PROPOSITION VII.

To reduce a Fraction *into its* Equivalent, *that shall have any assigned* Denominator.

RULE.

Multiply the Numerator of the Fraction by the assigned Denominator, and divide the Product by the Denominator of

of the Fraction; the Quotient ſhall be the Numerator required.

EXAMPLE I.

To reduce $\frac{3}{4}$ into a Fraction, whoſe Denominator is 12.

Thus $\frac{3\times12}{4}=\frac{36}{4}$, and $\frac{36}{4}=9$, then $\frac{9}{12}$ is the required Fraction, or $\frac{3}{4}=\frac{4)3\times12}{12}=\frac{4)36}{12}=\frac{9}{12}$.

For $4:3::12:\frac{12\times3}{4}=\frac{36}{4}=9$.

Therefore $\frac{3}{4}\left(=\frac{4)3\times12}{12}\right)=\frac{9}{12}$.

EXAMPLE II.

To reduce $\frac{11}{12}$ into a Fraction whoſe Denominator is 84.

Thus $\frac{11\times84}{12}=\frac{924}{12}$ and $\frac{924}{12}=77$, then $\frac{77}{84}$ is the required Fraction; or, much neater,

Thus $\frac{11}{12}=\frac{12)11\times84}{84}=\frac{12)924}{84}=\frac{77}{84}$

EXAMPLE III.

To reduce $\frac{7}{8}$ into a Fraction whoſe Denominator is 56.

Thus $\frac{7}{8}=\frac{8)7\times56}{56}=\frac{8)392}{56}=\frac{49}{56}$ the required Fraction.

COROLLARY.

By this *Propoſition*, *Fractions* are reduced into their known Parts of *Time*, *Meaſure*, *Weight*, *Coin*, &c. As alſo into *Decimals*, *Sexageſſimals*, &c. and the contrary.

Thus $\frac{3}{4}$£. $=\left(\frac{3\times20}{4}=\right)$ 15*s*.

And $\frac{2}{3}$ Deg. $=\left(\frac{2\times60}{3}=\right)$

Alſo $\frac{7}{11}$ Deg. $=38', 10''$, ″, *&c.*

For

For 1st. $\frac{7}{11}$ Deg. $=\left(\frac{7\times60}{11}=\right)$ $38'\frac{2}{11}$.

2d. $\frac{2}{11}$ Min. $=\left(\frac{2\times60}{11}=\right)$ $10''\frac{10}{11}$.

3d. $\frac{10}{11}$ Sec. $=\left(\frac{10\times60}{11}=\right)$ $54'''\frac{6}{11}$.

4th. $\frac{6}{11}$ Thirds $=\left(\frac{6\times60}{11}=\right)$ $32''''\frac{8}{11}$, *&c.*

SCHOLIUM.

Hence alſo, to reduce a *Fraction* into its *Equivalent*, that ſhall have any *aſſigned Numerator*.

EXAMPLE I.

To reduce $\frac{3}{4}$ into a *Fraction*, whoſe *Numerator* is 9.

Thus $\frac{3}{4}=\left(\frac{9}{3)4\times9}=\frac{9}{3)36}=\right)$ $\frac{9}{12}$.

For $3:4::9:\frac{9\times4}{3}=\frac{36}{3}$—12.

This Corollary is in Mr Jones's *excellent Treatiſe, intitled* Synopſis Palmariorum Matheſeos, *p.* 93.

EXAMPLE II.

To reduce $\frac{7}{8}$ into a *Fraction*, whoſe Numerator is 21.

Thus $\frac{7}{8}=\left(\frac{21}{7)8\times21}=\frac{21}{7)168}=\right)$ $\frac{21}{24}$ the required Fraction.

PROPOSITION VIII.

TO reduce Fractions *of different* Denominators *into their Equivalents, which ſhall have the ſame* Denominator.

RULE.

Multiply all the Denominators continually, for a common Denominator, and each Numerator continually by the other's Denominators for Numerators, that is, *all the Denominators multiplied continually together for a new Denominator, and each Numerator by all the Denominators, except its own, and the Products are the reſpective Numerators.*

Ex-

EXAMPLE I.

Reduce $\frac{2}{3}$ and $\frac{3}{4}$ into a common Denominator.

OPERATION.

Thus $\frac{2}{3}=\frac{8}{12}$, and $\frac{3}{4}=\frac{9}{12}$.

$\left.\begin{array}{l}\text{For } 3\times4=12 \text{ the Denominator common} \\ \text{And } 2\times4=8 \text{ a Numerator.}\end{array}\right\}\frac{8}{12}.$

$\left.\begin{array}{l}\text{Likewise } 3\times4=12 \text{ the common Denom. as before} \\ \qquad 3\times3=9 \text{ a Numerator, therefore}\end{array}\right\}\frac{9}{12}$

EXAMPLE II.

$\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$ reduced make $\frac{192}{240}$, $\frac{200}{240}$, $\frac{210}{240}$.

$\text{For}\left\{\begin{array}{l}5\times6=30\times8=240 \text{ Denominator common.} \\ 4\times6=24\times8=192, \text{ a Numerator, or } \frac{192}{240}. \\ 5\times5=25\times8=200, \text{ a Numerator, or } \frac{200}{240}. \\ 7\times5=35\times6=210, \text{ a Numerator, or } \frac{210}{240}.\end{array}\right.$

EXAMPLE III.

$\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{7}$, make $\frac{28}{56}$, $\frac{42}{56}$, $\frac{40}{56}$, or $\frac{14}{28}$, $\frac{21}{28}$, $\frac{20}{28}$.

$\text{For}\left\{\begin{array}{l}\frac{1}{2}=\frac{1\times4\times7}{2\times4\times7}=\frac{28}{56}=\frac{14}{28} \\ \frac{3}{4}=\frac{3\times2\times7}{4\times2\times7}=\frac{42}{56}=\frac{21}{28} \\ \frac{5}{7}=\frac{5\times2\times4}{7\times2\times4}=\frac{40}{56}=\frac{20}{28}\end{array}\right\}$ $\frac{14}{28}$, $\frac{21}{28}$ and $\frac{20}{28}$ are the required Fractions.

EXAMPLE IV.

$\frac{3}{5}$, $\frac{5}{7}$ and $\frac{8}{11}$ make $\frac{231}{385}$, $\frac{275}{385}$ and $\frac{280}{385}$.

$\text{For}\left\{\begin{array}{l}\frac{3}{5}=\frac{3\times7\times11}{5\times7\times11}=\frac{231}{385} \\ \frac{5}{7}=\frac{5\times5\times11}{7\times5\times11}=\frac{275}{385} \\ \frac{8}{11}=\frac{8\times5\times7}{11\times5\times7}=\frac{280}{385}\end{array}\right\}$ $\frac{231}{385}$, $\frac{275}{385}$, and $\frac{280}{385}$ are the respective Fractions.

Ex-

EXAMPLE V.

$\frac{3}{5}$, $\frac{5}{11}$, $\frac{6}{13}$, $\frac{11}{21}$, $\frac{17}{55}$ make $\frac{495495}{825825}$, $\frac{375375}{825825}$, $\frac{381150}{825825}$, $\frac{432575}{825825}$, $\frac{255255}{825825}$.

$$\text{For}\left\{\begin{array}{l}
\frac{3}{5}=\frac{3\times11\times13\times21\times55}{5\times11\times13\times21\times55}=\frac{495495}{825825}.\\
\frac{5}{11}=\frac{5\times5\times13\times21\times55}{11\times5\times13\times21\times55}=\frac{375375}{825825}\\
\frac{6}{13}=\frac{6\times5\times11\times21\times55}{13\times5\times11\times21\times55}=\frac{381150}{825825}\\
\frac{11}{21}=\frac{11\times5\times11\times13\times55}{21\times5\times11\times13\times55}=\frac{432575}{825825}\\
\frac{17}{55}=\frac{17\times5\times11\times13\times21}{55\times5\times11\times13\times21}=\frac{255255}{825825}
\end{array}\right\}\text{their respective Fractions.}$$

SCHOLIUM.

Hence, to find *two Integers*, that ſhall be one to the other as two given *Fractions*.

Suppoſe the Fractions were $\frac{2}{3}$ and $\frac{5}{7}$.

Then $\frac{2}{3} : \frac{5}{7} :: \frac{14}{21} : \frac{15}{21}$ by this Propoſition.

But $\frac{14}{21} : \frac{15}{21} :: 14 : 15$, by *Cor.* 3. *page* 14.

Therefore $\frac{2}{7} : \frac{5}{7} :: 14 : 15$.

PROPOSITION IX

TO reduce compound Fractions *into* ſingle *ones*.

RULE.

Multiply the Numerators continually for a new Numerator, and multiply the Denominators continually for a new Denominator.

EXAMPLE I.

Reduce $\frac{2}{3}$ of $\frac{3}{4}$ into a ſingle Fraction.

Thus $2\times3=6$ the Numerator
And $3\times4=12$ the Denominator } $\frac{6}{12}=\frac{1}{2}$ the required Fraction.

EXAMPLE II.

Reduce $\frac{2}{3}$ of $\frac{4}{7}$ into a ſingle Fraction.

Thus

Thus $\frac{2\times 4}{3\times 7}=\frac{8}{21}$.

For if $\frac{4}{7}$ be $=$A, then $\frac{2}{3}$ of $\frac{4}{7}$ is $\frac{2}{3}A=\frac{2}{3}\times A$.

Therefore $\frac{2}{3}$ of $\frac{4}{7}=\frac{2\times 4}{3\times 7}=\frac{8}{21}$.

EXAMPLE III.

Reduce $\frac{2}{5}$ of $\frac{4}{9}$ of $\frac{5}{11}$ of $\frac{16}{21}$ of $\frac{81}{95}$ into a ſingle Fraction.

Thus $\frac{2\times 4\times 5\times 16\times 81}{5\times 9\times 11\times 21\times 95}=\frac{51840}{987525}=\frac{1920}{36575}$.

PROPOSITION X.

TO reduce a Fraction from one Denominator to another.
And this is either Aſcending or Deſcending. 1. Aſcending.

EXAMPLE I.

Reduce $\frac{4}{5}$ of a Penny to a Fraction of a Pound.
Thus $\frac{4}{5}$ of $\frac{1}{12}$ of $\frac{1}{20}$. For 1*d.* is the $\frac{1}{12}$ of $\frac{1}{20}$.
Therefore $\frac{4}{5}$ is the $\frac{4}{5}$ of $\frac{1}{12}$ of $\frac{1}{20}$.

Then $\frac{4\times 1\times 1}{5\times 12\times 20}=\frac{4}{1200}=\frac{1}{300}$ the required Fraction.

EXAMPLE II.

Reduce $\frac{13}{49}$ of a Penny into the Fraction of a Pound Sterling.

For the ſame Reaſons as above, that 1*d.* is the $\frac{1}{12}$ of $\frac{1}{20}$ of a Pound Sterling.

So is $\frac{13}{49}$ of $\frac{1}{12}$ of $\frac{1}{20}$.

Then $\frac{13\times 1\times 1}{49\times 12\times 20}=\frac{13}{11760}$ the required Fraction.

EXAMPLE III.

Reduce $\frac{1}{7}$ of an Ounce *Averdupois Weight* into the Fraction of an Hundred Weight.

Thus $\frac{1}{7}$ of $\frac{1}{16}$ of $\frac{1}{112}$ of a Hundred.

Then $\frac{1\times 1\times 1}{7\times 16\times 112}=\frac{1}{12544}$, the required Fraction.

EXAMPLE IV.

Reduce $\frac{5}{9}$ of a Penny Weight *Troy* into the Fraction of a Pound *Troy*.

Thus $\frac{5}{9}$ of $\frac{1}{20}$ of $\frac{1}{12}$ of a Pound.

Then $\frac{5\times1\times1}{9\times20\times12}=\frac{5}{2160}=\frac{1}{432}$.

SCHOLIUM.

2. Hence we may reduce a descending Fraction.

EXAMPLE I.

Reduce $\frac{4}{1200}$ of a Pound into the Fraction of a Penny.

Thus $4\times20\times12=960$.

And $\frac{960}{1200}=\frac{4}{5}$ the required Fraction.

EXAMPLE II.

Reduce $\frac{13}{11760}$ of a Pound into a Fraction of a Penny.

Thus $13\times20\times12=3120$.

And $\frac{3120}{11760}=\frac{13}{49}$ the required Fraction. As is manifestly evident, being the converse of *Example* 2. of this *Proposition*.

PROPOSITION XI.

TO find the Value of a Vulgar Fraction.

RULE.

Multiply the Numerator by the Parts of the next inferior Denomination, and divide the Product by the Denominator, the Quotient gives the Value in the Parts you multiplied by; and if after this Division any Thing remain, multiply that Remainder by the next inferior Denomination, dividing the Product by the Denominator as before; and so proceed till you can bring it no lower, and if then any Thing remain, set it for a Numerator over the former Denominator.

EXAMPLE I.

What is the Value of $\frac{6}{8}$ of a Pound Sterling?

Thus

Thus $\frac{6\times20}{8}=\frac{120}{8}=15s.$ the required Value.

EXAMPLE II.

What is the $\frac{17}{19}$ of a Pound Sterling.

Thus $\frac{17\times20}{19}=\frac{340}{19}$.

And 19)340(17 *Shillings*
19
150
133
17 Remainder
12 next inferior Denomination
19)204(10 *Pence*
19
14
4 next (or lowest) inferior Denomination
19)56(2 *Farthings.*
38
18

So $\frac{17}{19}l.=17s.\ 10d.\ \frac{1}{2}\ \frac{18}{19}$.

EXAMPLE III.

What is the Value of $\frac{3}{5}$ of a Shilling.

Thus 3
12 Pence
5)36(7$d.\ \frac{1}{5}$ or $\frac{3}{5}s.=7d.\ \frac{1}{5}$.
35
1

EXAMPLE IV.

What is the Value of $\frac{7}{9}$ of a Penny ?

Thus 7
4
9)28(3*far.* $\frac{1}{9}$.
27
1 so $\frac{7}{9}d.=\frac{3}{4}d.\ \frac{1}{9}$.

EXAMPLE V.

What is the Value of $\frac{2}{9}$ of a Pound *Troy*?

Thus
```
     2
    12
 9)24(2
   18
    6
   20
9)120(13
  117
    3
   24
  9)72(8
    72
```

So that $\frac{2}{9}$*lb. Troy* = 2*oz.* 13*pw.* 8*gr.*

EXAMPLE VI.

What is the $\frac{18}{29}$ of 11*C.* 3*qr.* 16*lb.* Averdupois Weight?

Thus
```
      C.  qr.  lb.
      11   3   16
        4
       47
       28
      382
       95
     1332
       18
    10656
     1332      lb.          C. qr. lb.
29)23976(826 22/29 = 7  1  14 22/29, the required
        22 Remainder.                       [Value
```

EXAMPLE VII.

What is the Value of $\frac{5}{6}$ of 13*l.* 18*s.* 6*d.* $\frac{1}{2}$?

Thus

	l.	*s.*	*d.*
Thus	13	18	$6\frac{1}{2}$
			5
6)	69	12	$8\frac{1}{2}$
£.	11	12	$1\frac{1}{4}\frac{2}{3}$

EXAMPLE VIII.

What is the $\frac{11}{12}$ of 19*C.* 2*qr.* 11*lb.*

	C.	*qr.*	*lb.*	
Thus	19	2	11	
			11	
12)	215	2	9	
	17	3	$24\frac{1}{12}$	the required Value.

EXAMPLE IX.

What Part of a Pound Sterling is $\frac{2}{3}$ of $\frac{5}{9}$?

First $\frac{2}{3}$ of $\frac{5}{9} = \frac{10}{27}$. For $\frac{2\times5}{3\times9} = \frac{10}{27}$ by *Proposition* 9.

Therefore $\frac{2}{3}$ of $\frac{5}{9} = \frac{10}{27}$.

EXAMPLE X.

What is the $\frac{5}{8}$ of 13*d.* $\frac{1}{2}$? By observing the Operation as in the above it is 8*d.* 0*q.* $\frac{6}{8}$. I think from what I have here delivered, it will be easy for any one to find the Value of any Fraction; and the Reason of my dwelling so much on this last Proposition, is, I have found it to be of good Use to Beginners.

PROPOSITION XII.

TO find of what Whole any proposed contract Number is any Part or Parts?

RULE.

Suppose the Fraction inverted, and then work as in the last Proposition.

EXAMPLE I.

What is 15*s.* the $\frac{3}{4}$ of?

Thus

Thus $\frac{15 \times 4}{3} = \frac{60}{3} = 20s.$

So 15*s.* is the $\frac{3}{4}$ of 20*s.*

EXAMPLE II.

What is 17*s.* 6*d.* the $\frac{7}{8}$ of?

$$\begin{array}{rrrr} & & s. & d. \\ & & 17 & 6 \\ & & & 8 \\ \hline 7) & 7 & 00 & 0 \\ \hline £. & 1 & 00 & 0 \\ \hline \end{array}$$

So that 17*s.* 6*d.* is the $\frac{7}{8}$ of 1*l.*

EXAMPLE III.

What is 3*C.* 2*q.* 5*lb.* 13*oz.* $\frac{1}{3}$ the $\frac{11}{24}$ of?

$$\begin{array}{lrrrr} & C. & q. & lb. & oz. \\ \text{Thus} & 3 & 2 & 5 & 13\frac{1}{3} \\ & & & & 6 \\ \hline & 21 & 1 & 7 & 0 \\ & & & & 4 \\ \hline 11) & 85 & 1 & 0 & 0 \\ \hline & 7 & 3 & 0 & 0 \\ \hline \end{array}$$

for $\frac{1}{3} \times \frac{6}{1} = \frac{6}{3} = 2.$

7 3 0 0 the Anſwer.

EXAMPLE IV.

What is the 3*q.* 1*lb.* 5*oz.* $\frac{7}{21}$ the $\frac{16}{21}$ of?

$$\begin{array}{lrrrr} & & q. & lb. & oz. \\ \text{Thus} & & 3 & 1 & 5\frac{7}{21} \\ & & & & 7 \\ \hline & 5 & 1 & 9 & 5\frac{7}{21} \\ & & & & 3 \\ \hline 16) & 16 & 0 & 0 & 0 \\ \hline & 1 & 0 & 0 & 0 \\ \hline \end{array}$$

So that 3*q.* 1*lb.* 5*oz.* $\frac{7}{21}$ is the $\frac{16}{21}$ of a Hundred Weight.

PRO-

PROPOSITION XIII.

TO find what Part or Parts any contract Number is of any other of the same Kind.

RULE.

Let both the Numbers be reduced to the least Name found, and make the lesser the Numerator, and the greater the Denominator of a Fraction, which so done reduce it to its least Name.

EXAMPLE I.

What Part of 4*l.* 6*s.* 8*d.* is 2*l.* 3*s.* 4*d?*

	l.	*s.*	*d.*		
Thus	2	3	4	= 520	and $\frac{520}{1040} = \frac{1}{2}$.
And	4	6	8	= 1040	

Therefore 2 3 4 is $\frac{1}{2}$ of 4*l.* 6*s.* 8*d.* as is self-evident.

EXAMPLE II.

What Part of 11*l.* 11*s.* 10*d.* is 6*l.* 13*s.* 8*d*?

	l.	*d.*	*d.*	*d.*	
Thus	11	11	10	= 2782	$= \frac{802}{1391}$.
And	6	13	8	= 1604	

Therefore $\frac{802}{1391}$ is the Answer.

COROLLARY.

Hence might the Parts be found in Measure, Weights, Quantities, Time, &c.

ADDI-

ADDITION.

PROPOSITION XIV.

If the Denominators are not equal, they muſt be reduced to ſuch as have equal ones; then by this

RULE.

The Sum of the Numerators ſet over the common Denominator ſhall be the Sum of the given Fractions.

DEMONSTRATION.

As the Denominators contain the Number of Units the Numerators have, (by *Definition* 15, *pag.* 4.) ſo the Numerators are only to be added; and becauſe that cannot be done, except they be homogeneous, (by *Definition* 9, *pag.* 3.) ſo they are brought to the ſame Denominator.

CASE I. EXAMPLE I.

Add $\frac{1}{2}$ and $\frac{1}{3}$.

Thus $2\times3=6$ the Denominator.
$2\times1=2$ a Numerator.
$3\times1=3$ a Numerator.

Then $\frac{2+3}{6}=\frac{5}{6}$ the Sum.

EXAMPLE II.

Add $\frac{3}{4}$ to $\frac{4}{5}$.

Thus $4\times5=20$ Denominator.
$3\times5=15$ Numerator.
$4\times4=16$ Numerator.

Then $\frac{15+16}{20}=\frac{31}{20}=1\frac{11}{20}$ the Sum.

EXAMPLE III.

Add $\frac{2}{7}$, $\frac{5}{11}$ and $\frac{19}{31}$.

Thus

Thus $7\times11\times31=2387$ a common Denominator.
$2\times11\times31=682$ a Numerator.
$5\times7\times31=1085$ a Numerator.
$19\times7\times11=1463$ a Numerator.

Therefore $\frac{682+1085+1463}{2387}=\frac{3230}{2387}=1\frac{443}{2387}$ the required Sum.

EXAMPLE IV.

Add $\frac{113}{415}$ to $\frac{3119}{5326}$.

Thus $415\times5326=2210290$ Denominator.
$415\times3119=1294385$ a Numerator.
$113\times5326=601838$ a Numerator.

Therefore $\frac{601838+1294385}{2210290}=\frac{1896223}{2210290}$ the required Sum.

CASE II. EXAMPLE V.

Add 13 to $21\frac{11}{13}$.

Thus $13+21\frac{11}{13}=34\frac{11}{13}$ the required Sum.

EXAMPLE VI.

Add 544 to $261\frac{34198}{57119}$.

Thus $544+261\frac{34198}{57119}=805\frac{34198}{57119}$ the required Sum.

CASE III. EXAMPLE VII.

Add $\frac{3}{11}$ to 47.

Thus $47+\frac{3}{11}=47\frac{3}{11}$ the Sum required.

EXAMPLE VIII.

Add $\frac{414}{686}$ to 429.

Thus $429+\frac{31}{49}=429\frac{31}{49}=429\frac{414}{686}$ the required Sum.

CASE IV. EXAMPLE IX.

Add $6\frac{1}{2}$ to $34\frac{3}{5}$.

Thus $6+34=40$.

And $\frac{1}{2}+\frac{3}{5}=\frac{6+5}{10}=\frac{11}{10}=1\frac{1}{10}$.

Therefore $40+1\frac{1}{10}=41\frac{1}{10}$ the required Sum.

Ex-

EXAMPLE X.

Add $27\frac{7}{9}$ to $31\frac{11}{21}$.

Thus $27+31=58$.

And $\frac{7}{9}+\frac{11}{21}=\frac{99+147}{198}=\frac{246}{189}=1\frac{57}{189}$, by *Case* 1.

Therefore $58+1\frac{57}{189}=59\frac{57}{189}$ the Sum required.

EXAMPLE XI.

Add $28\frac{2}{7}$, $59\frac{5}{11}$ and $64\frac{19}{31}$.

Thus $28+59+64=151$.

And $\frac{2}{7}+\frac{5}{11}+\frac{19}{31}=1\frac{443}{2387}$, by *Ex.* 3. *Case* 1.

Therefore $151+1\frac{443}{2387}=152\frac{443}{2387}$ the required Sum.

EXAMPLE XII.

$$\text{Add}\quad \begin{array}{r} 871\ \frac{3}{5} \\ 94\ \frac{5}{11} \\ 4621\ \frac{6}{13} \\ \frac{11}{21} \\ 432\ \frac{17}{55} \end{array}$$

Thus $871+94+4621+432=6018$.

And $\frac{3}{5}+\frac{5}{11}+\frac{6}{13}+\frac{11}{21}+\frac{17}{55}$ reduced to a common Denominator, by *Proposition* 8, *Ex.* 5. is $\frac{495495}{825825}+\frac{375375}{825825}+\frac{381150}{825825}+\frac{432575}{825825}+\frac{255255}{825825}=\frac{1939850}{825825}=2\frac{288200}{825825}$ and $2\frac{288200}{825825}+6018=6020\frac{288200}{825825}$ the required Sum.

I have insisted the longer on this Case, because it is most in Practice, and some Care to be had in the Reduction, which I would advise the Reader to be well acquainted with before he proceed to Subtraction; he will then find no Difficulty, but what he will easily surmount.

CASE V. EXAMPLE XIII.

Add $\frac{3}{11}$, $\frac{4}{11}$, $\frac{5}{11}$, $\frac{6}{11}$, $\frac{7}{11}$, $\frac{9}{11}$.

This requires only the Numerators to be added, for the Denominators are all a-like, and to the Sum of the Numerators subscribe the Denominator.

Thus $\frac{3+4+5+6+7+9}{11}=\frac{34}{11}=3\frac{1}{11}$ the required Sum.

Ex-

EXAMPLE XIV.

Add $\frac{18}{131}$, $\frac{21}{131}$, $\frac{53}{131}$, $\frac{116}{131}$.

Thus $\frac{18+21+53+116}{131}=\frac{208}{131}=1\frac{77}{131}$.

EXAMPLE. XV.

Add $\frac{2}{9}$ to $\frac{5}{9}$.

Then $\frac{5}{9}+\frac{2}{9}=\frac{5+2}{9}=\frac{7}{9}$.

For $\frac{5}{9}:\frac{2}{9}::5:2$.
And $\frac{5}{9}+\frac{2}{9}:\frac{2}{9}::5+2:2$.
But $\frac{5+2}{9}:\frac{2}{9}::5+2:2$.

Therefore $\frac{5}{9}+\frac{2}{9}:\frac{2}{9}::\frac{5+2}{9}:\frac{2}{9}$.

Consequently $\frac{5}{9}+\frac{2}{9}=\frac{5+2}{9}=\frac{7}{9}$.

This is the *Demonstration* why there must be a common Denominator before Addition can be had.

CASE VI. EXAMPLE XVI.

Add $\frac{14}{27}$ to $\frac{3}{4}$ of $\frac{5}{6}$.

First $\frac{3}{4}$ of $\frac{5}{6}$ must be reduced to a single Fraction, by *Proposition* 9. and it is $\frac{15}{24}=\frac{5}{8}$.

Then $\frac{14}{27}+\frac{5}{8}=\frac{112+135}{216}=\frac{247}{216}=1\frac{31}{216}$, by *Ex.* 1. *Case* 1.

EXAMPLE XVII.

Add $\frac{131}{251}$ to $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$.

First $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}=\frac{30}{72}$, by *Prop.* 9.

And $\frac{131}{251}+\frac{30}{72}=\frac{7530+9432}{18072}=\frac{16962}{18072}$, by *Case* 1.

CASE VII. EXAMPLE XVIII.

Add $\frac{4}{5}$ of a Pound to $\frac{7}{8}$ of a Shilling.

Thus $\frac{4}{5}$ of a Pound is $=\frac{4}{5}$ of $\frac{1}{20}$.

And $\frac{4\times20}{1\times5}=\frac{80}{5}=\frac{16}{1}$, by *Scholium* to *Prop.* 8.

Therefore $\frac{7}{8}+\frac{16}{1}=\frac{128+7}{8}=\frac{135}{8}=16\frac{7}{8}$, the required Sum.

EXAMPLE XIX.

Add $\frac{3}{4}l.$ to $\frac{5}{6}$ of a Shilling.

Thus $\frac{5}{6}$ of a Shilling is $=\frac{5}{6}$ of $\frac{1}{20}$.
Then $\frac{5}{6}$ of $\frac{1}{20}=\frac{5}{120}l.$ and

Therefore $\frac{5}{120}+\frac{3}{4}=\frac{360+20}{480}=\frac{380}{480}=\frac{19}{24}l.$ the required sum.

It would have been the same if we had reduc'd $\frac{3}{4}l.$ to the Fraction of a Shilling, by the *Scholium* to *Prop.* 8. which is found to be $=\frac{60}{4}s.$

For $\frac{3}{4}$ of $\frac{1}{20}=\frac{3\times20}{4}=\frac{60}{4}$ and $\frac{60}{4}+\frac{5}{6}=\frac{360+20}{24}=\frac{380}{24}=15s.\ \frac{5}{6}=15s.\ 10d.$
Therefore $\frac{19}{24}l.=15s.\ \frac{5}{6}=15s.\ 10d.$

CASE VIII. EXAMPLE XX.

Add $\frac{3}{4}$ of $\frac{5}{6}$ to $\frac{3}{5}$ of $\frac{7}{8}$.

Thus $\frac{3}{4}$ of $\frac{5}{6}=\frac{15}{24}$, and $\frac{3}{5}$ of $\frac{7}{8}=\frac{21}{40}$.
Then $\frac{15}{24}+\frac{21}{40}=\frac{1104}{960}=1\frac{3}{20}$ the required sum.

EXAMPLE XXI.

Add $\frac{2}{3}$ of $\frac{5}{11}$ of $\frac{7}{19}$ to $\frac{21}{73}$ of $\frac{12}{13}$.

Thus $\frac{2}{3}$ of $\frac{5}{11}$ of $\frac{7}{19}=\frac{70}{627}$; and $\frac{21}{73}$ of $\frac{12}{13}=\frac{252}{949}$, by *Prop.* 9. Then by *Case* 1. $\frac{70}{627}+\frac{252}{949}=\frac{224434}{595023}$.

CASE IX. EXAMPLE XXII.

Add 24 and $12\frac{1}{2}$ of $\frac{2}{3}$.

Thus $24+12=36$.

And

And $\frac{1}{2}$ of $\frac{2}{3} = \frac{1 \times 2}{2 \times 3} = \frac{2}{6} = \frac{1}{3}$.

Therefore $36 + \frac{1}{3} = 36\frac{1}{3}$ the Sum.

EXAMPLE XXIII.

Add 19 and $7\frac{1}{2}$ of $\frac{2}{7}$ together.

Thus $19 + 7 = 26$.
And $\frac{1}{2}$ of $\frac{2}{7} = \frac{2}{14} = \frac{1}{7}$.
Then $26\frac{1}{7}$ is the Sum.

CASE X. EXAMPLE XXIV.

Add $\frac{1}{3}$ of 95 and $\frac{7}{8}$ of 14 together.

Thus $\frac{1}{3}$ of $\frac{95}{1} = \frac{95}{3}$.
And $\frac{7}{8}$ of $\frac{14}{1} = \frac{98}{8}$.
Then $\frac{95}{3} + \frac{98}{8} = \frac{1054}{24} = 43\frac{22}{24} = 43\frac{11}{12}$, the Sum.

EXAMPLE XXV.

Add $\frac{11}{12}$ of 100 and $\frac{3}{4}$ of 105 together.

Thus $\frac{11}{12}$ of $\frac{100}{1} = \frac{1100}{12}$.
And $\frac{3}{4}$ of $\frac{105}{1} = \frac{315}{4}$.
Therefore $\frac{1100}{12} + \frac{315}{4} = \frac{8180}{48} = 170\frac{5}{12}$ the Sum.

Here the Reader may obſerve, that iſ any of the Fractions to be added be compound ones, they muſt be reduced to ſimple Fractions, which is eaſily done by the Rules of Reduction, and then to be added.

I ſhall only give another Example or two, in a Method ſomething different from what I have laid down, and then make a Tranſition to Subtraction.

PROPOSITION XV.

TO add to any Fraction any Part or Parts of the ſame Fraction.

RULE.

Add the Denominator to the Numerator of the Fraction expreſſing the Part or Parts to be added, and that Sum multiply by the other Numerator, and this ſhall be a new

 Numerator;

Numerator; then multiply the two Denominators together, and the Product is a new Denominator.

EXAMPLE I.

To $\frac{3}{5}$ add $\frac{4}{9}$ of the same $\frac{3}{5}$.

Thus $4+9=13$, and $13\times3=39$ Numerator.
And $5\times9=45$ Denominator.
Therefore $\frac{39}{45}=\frac{13}{15}$ the required Sum.

EXAMPLE II.

To $\frac{11}{17}$ add $\frac{4}{5}$ of the same $\frac{11}{17}$.

Thus $4+5=9$, and $9\times11=99$ Numerator.
And $17\times5=85$ Denominator.
Therefore $\frac{99}{85}=1\frac{14}{85}$ the required Sum.

EXAMPLE III.

To $\frac{3}{4}$*l.* add $\frac{3}{5}$ of the same $\frac{3}{4}$.

Thus $5+3=8$, and $8\times3=24$ Numerator.
And $4\times5=20$ Denominator.
Therefore $\frac{24}{20}=1$*l.* $\frac{1}{5}=1$*l.* 4*s.*
For 15*s.* + 9*s.* = 24*s.* = 1*l.* 4*s.*

SUBTRACTION.

PROPOSITION XVI.

IF the Denominators are not equal, they muſt be reduced to ſuch as have equal ones, managed as in Addition; then by this

RULE.

The Difference of the Numerators, ſet over the common Denominator, ſhall be the Difference of the given Fractions.

CASE I. EXAMPLE I.

Let $\frac{2}{9}$ be taken from $\frac{5}{9}$.

Then $\frac{5}{9} - \frac{2}{9} = \frac{3}{9} = \frac{1}{3}$.

DEMONSTRATION.

For $\frac{5}{9} : \frac{2}{9} :: 5 : 2$, by *Cor.* 3. *pag.* 14.

And $\frac{5}{9} - \frac{2}{9} : \frac{2}{9} :: 5-2 : 2$.

But $\frac{5-2}{9} : \frac{2}{9} :: 5-2 : 2$.

Therefore $\frac{5}{9} - \frac{2}{9} : \frac{2}{9} :: \frac{5-2}{9} : \frac{2}{9}$.

Conſeq. $\frac{5}{9} - \frac{2}{9} = \frac{5-2}{9} = \frac{3}{9} = \frac{1}{3}$.

CASE II. EXAMPLE II.

From $\frac{5}{6}$ take $\frac{1}{3}$.

Thus $6 \times 3 = 18$ Denominator.
$5 \times 3 = 15$ Numerator.
$6 \times 1 = 6$ Numerator.

Then $\frac{15-6}{18} = \frac{9}{18} = \frac{1}{2}$, as is evident from *Example* 1. in Addition.

EXAMPLE III.

Take $\frac{3}{7}$ from $\frac{5}{9}$.

Thus $7\times9=63$ Denominator.
$7\times5=35$ a Numerator.
$3\times9=27$ a Numerator.

Then $\frac{35-27}{63}=\frac{8}{63}$, Anſwer.

CASE III. EXAMPLE IV.

From $34\frac{11}{13}$ take 13.

Thus $34-13=21$.
And $\frac{11}{13}-0=\frac{11}{13}$.

Therefore $21\frac{11}{13}$ is the required Remainder, as is evident by Inſpection, and *Example* 5. in Addition being a Proof thereof.

EXAMPLE V.

From $29\frac{5}{11}$ take 18.

Thus $29\frac{5}{11}-18=11\frac{5}{11}$ the required Remainder.

CASE IV. EXAMPLE VI.

From 47 take $\frac{3}{11}$.

Thus $11-3=\frac{8}{11}$ Numerator.
And $47-1=46$.
Then $46\frac{8}{11}$ is the required Remainder.

Note, When you would Subtract a Vulgar Fraction from an Integer or whole Number; firſt ſubtract the Numerator from its Denominator, and the Remainder is a new Numerator to its Denominator; then carry 1 to be taken from the whole Number, and the Remainder is the required Fraction. An Example or two will render this very familiar.

EXAMPLE VII.

From 67 take $\frac{13}{19}$.

Thus $19-13=\frac{6}{19}$ Numerator.
And $67-1=66$.
Therefore $66\frac{6}{19}$ is the required Remainder.

Ex-

EXAMPLE VIII.

From 429 take $\frac{31}{49}$.

First $49-31=\frac{18}{49}$.

And $429-1=428$.

Therefore $428\frac{18}{49}$ is the required Remainder.

EXAMPLE IX.

From $41\frac{1}{10}$ take $34\frac{3}{5}$.

Thus $41-34=7$.

And $\frac{1}{10}-\frac{3}{5}=\frac{30-5}{50}=\frac{25}{50}=\frac{1}{2}$.

Therefore $7-\frac{1}{2}=6\frac{1}{2}$, the required Fraction, as is evident from *Ex*. 9. in Addition.

EXAMPLE X.

From $14\frac{4}{5}$ take $12\frac{2}{3}$.

Thus $14-12=2$.

And $\frac{4}{5}-\frac{2}{3}=\frac{12-10}{15}=\frac{2}{15}$.

Therefore $2\frac{2}{15}$ is the required Remainder.

EXAMPLE XI.

From $81\frac{5}{9}$ take $49\frac{3}{7}$.

Thus $81-49=32$.

And $\frac{5}{9}-\frac{3}{7}=\frac{35-27}{63}=\frac{8}{63}$.

Therefore $32+\frac{8}{63}=32\frac{8}{63}$, the required Remainder.

CASE VI. EXAMPLE XII.

From $13\frac{1}{3}$ take $\frac{3}{8}$.

Thus $\frac{1}{3}-\frac{3}{8}=\frac{9-8}{24}=\frac{1}{24}$.

Then $13-\frac{1}{24}=12\frac{23}{24}$ is the required Remainder, as is manifest from the *Note* in *Ex*. 6. above.

CASE

CASE VII. EXAMPLE XIII.

From $\frac{13}{14}$ take $\frac{2}{3}$ of $\frac{8}{9}$.

Thus $\frac{2}{3}$ of $\frac{8}{9}=\frac{16}{27}$.

Then $\frac{13}{14}-\frac{16}{27}=\frac{351-224}{378}=\frac{127}{378}$ the required Remainder.

CASE VIII. EXAMPLE XIV.

Take $\frac{3}{4}$ of $\frac{5}{9}$ from $\frac{2}{3}$ of $\frac{3}{5}$.

First $\frac{3}{4}$ of $\frac{5}{9}=\frac{15}{36}$.
And $\frac{2}{3}$ of $\frac{3}{5}=\frac{6}{15}$.

Then $\frac{6}{15}-\frac{15}{36}=\frac{225-216}{540}=\frac{9}{540}=\frac{3}{180}=\frac{1}{60}$ the required Remainder.

EXAMPLE XV.

From $\frac{4}{5}$ of $\frac{6}{7}$ take $\frac{7}{8}$ of $\frac{4}{11}$

First $\frac{4}{5}$ of $\frac{6}{7}=\frac{24}{35}$.
And $\frac{7}{8}$ of $\frac{4}{11}=\frac{28}{88}$.

Then $\frac{24}{35}-\frac{28}{88}=\frac{2112-980}{3080}=\frac{1132}{3080}=\frac{283}{770}$ the required Remainder.

CASE IX. EXAMPLE XVI.

From $\frac{3}{4}$ and $\frac{2}{3}$ take $\frac{7}{8}$ and $\frac{9}{10}$.

First $\frac{3}{4}$ and $\frac{2}{3}=\frac{9+8}{12}=\frac{17}{12}$.

And $\frac{7}{8}$ and $\frac{9}{10}=\frac{72+70}{80}=\frac{142}{80}$.

Then $\frac{17}{12}-\frac{142}{80}=\frac{1704-1360}{960}=\frac{344}{960}=\frac{43}{120}$, the required Remainder.

EXAMPLE XVII.

From $\frac{1}{3}$ and $\frac{3}{7}$ take $\frac{3}{5}$ and $\frac{4}{11}$.

Thus

Thus $\frac{1}{3}$ and $\frac{3}{7} = \frac{9+7}{21} = \frac{16}{21}$.

And $\frac{3}{5}$ and $\frac{4}{11} = \frac{33+20}{55} = \frac{53}{55}$.

Then $\frac{16}{21} - \frac{53}{55} = \frac{1113-880}{1155} = \frac{233}{1155}$ the required Remainder.

If theſe Examples be well underſtood, the whole Buſineſs of *adding* and *ſubtracting Vulgar Fractions* will be eaſy; which I think really much more difficult to perform than either Multiplication or Diviſion, as will appear by the next Propoſitions.

MULTIPLICATION.

PROPOSITION XVII.

To multiply one Fraction by another, multiply the Numerators into one another, and likewise the Denominators, thus $\frac{2}{3} \times \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$.

DEMONSTRATION.

This is also Division; for to multiply $\frac{2}{3}$ by $\frac{1}{2}$, is the same as to take the half of $\frac{2}{3}$ Parts; and therefore if $\frac{4}{5}$ be multiplied by $\frac{2}{3}$, we must find 2 third Parts of 4 Fifths; then $\frac{4}{5}$ being the Dividend, it must be divided into as many equal Parts as the Denominator of the Multiplier hath Unities, *viz.* 3, and that Part must be multiplied by the Numerator 2. Again, if $\frac{3}{4}$ be multiplied by $\frac{2}{3}$ it will will be $\frac{4 \times 2}{4 \times 3} = \frac{8}{12} = \frac{2}{3}$ by the 2d following *Lemma*.

And $\frac{6}{12} : \frac{8}{12} :: 6 : 8$ by *Corollary* 3. *pag.* 14.
That is $\frac{6}{12} : \frac{2}{3} :: 3 : 4$.
Or, $\frac{6}{12} : \frac{2}{3} :: \frac{3}{4} : 1$.
Therefore $\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$ by nature of Proportion.

SCHOLIUM I.

Seeing this is Division upon the Matter, it appears why the Product becomes less, therefore the Product of two proper Fractions must be less than either of them. Thus $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$, and $\frac{10}{21}$ is less than either $\frac{2}{3}$ or $\frac{5}{7}$.
For $1 : \frac{5}{7} :: \frac{2}{3} : \frac{10}{21}$, by the Nature of Multiplication.

SCHOLIUM II.

The Product of any Quantity, multiplied by a proper Fraction, is always less than that Quantity.

For in multiplying by an Unit, the Product will be equal to the Multiplicand,

But

But a leſs Multiplier gives a leſs Product.

Therefore multiplying by a proper Fraction (*i. e.* by leſs than Unit) the Product muſt be leſs than the Multiplicand.

Note, If mixt Fractions are to be multiplied together, reduce them to improper ones.

If compound Fractions are to be multiplied together, reduce them to ſingle ones.

If one of the Factors be a whole Number, it muſt be made an improper Fraction.

But before I give Examples in multiplying Fractions one by another, it will be neceſſary, in order to give the Learner a clear Idea of the Reaſon and Certainty of ſuch Rules, to premiſe the two following *Lemmas*.

LEMMA I.

To multiply the Numerator, is to multiply the Fraction; thus $\frac{2\times 3}{5} = \frac{2}{5}\times 3$.

For $5 : 2 :: 3 : \frac{2\times 3}{5}$.

And $5 : 2 :: 1 : \frac{2}{5}$.

Therefore $1 : \frac{2}{5} :: 3 : \frac{2\times 3}{5}$.

But $1 : \frac{2}{5} :: 3 : \frac{2}{5}\times 3$.

Therefore $\frac{2\times 3}{5} = \frac{2}{5}\times 3$.

LEMMA II.

The Terms of a Fraction being multiplied or divided by the ſame Quantity alter not its Value.

Thus $\frac{2\times 3}{2\times 4} = \frac{6}{8}$.

For $2\times 3\ (=6) : 2\times 4\ (=8) :: 3 : 4$.

Therefore $\frac{3}{4} = \frac{6}{8}$.

Alſo $\frac{5)10(2}{5)15(3} = \frac{2}{3}$.

For $\frac{10}{5} : \frac{15}{5} :: 10 : 15$, by *Corollary* 3. *pag.* 14.

But $\frac{10}{5} : \frac{15}{5} :: 2 : 3$.

Therefore $\frac{10}{15} = \frac{2}{3}$.

Seeing

Seeing (I hope) I have fully explained the Nature of Multiplication, and demonstrated its Rules sufficiently, it will be proper now to give some Examples.

CASE I. EXAMPLE I.

$\frac{3}{5} \times \frac{2}{3}$

Thus $\frac{3}{5} \times \frac{2}{3} = \frac{6}{15} = \frac{2}{5}$ the required Product.

EXAMPLE II.

Multiply $\frac{11}{17}$ by $\frac{18}{37}$.

Thus $11 \times 18 = 198$.

And $17 \times 37 = 629$.

Therefore $\frac{11 \times 18}{17 \times 37} = \frac{198}{629}$ the required Product.

CASE II. EXAMPLE III.

Multiply 12 by $3\frac{3}{5}$.

Thus $3\frac{3}{5} = \frac{\overline{3 \times 5 + 3}}{5} = \frac{18}{5}$, by *Prop.* 5.

Then $\frac{12 \times 18}{1 \times 5} = \frac{216}{5} = 43\frac{1}{5}$ the required Product.

EXAMPLE IV.

Multiply 171 by $56\frac{35}{71}$.

Thus $56\frac{35}{71} = \frac{\overline{56 \times 71 + 35}}{71} = \frac{4011}{71}$.

Then $\frac{171 \times 4011}{1 \times 71} = \frac{685881}{71} = 9660\frac{21}{71}$ the required Product.

CASE III. EXAMPLE V.

Multiply 11 by $\frac{5}{7}$.

Thus $\frac{11 \times 5}{1 \times 7} = \frac{55}{7} = 7\frac{6}{7}$ the required Product.

Ex-

EXAMPLE VI.

Multiply 151 by $\frac{112}{121}$.

Thus $\frac{151\times112}{1\times121}=\frac{16912}{121}=139\frac{93}{121}$ the required Product.

CASE IV. EXAMPLE VII.

Multiply $17\frac{5}{9}$ by 47.

Thus $17\frac{5}{9}=\frac{\overline{17\times9}+5}{9}=\frac{158}{9}$.

And $\frac{158\times47}{9\times1}=\frac{7426}{9}=825\frac{1}{9}$ the required Product.

EXAMPLE VIII.

Multiply $51\frac{23}{27}$ by 13.

Thus $51\frac{23}{27}=\frac{\overline{51\times27}+23}{27}=\frac{1400}{27}$.

And $\frac{13\times1400}{1\times27}=\frac{18200}{27}=674\frac{2}{27}$, the required Product.

CASE V. EXAMPLE IX.

Multiply $11\frac{1}{3}$ by $3\frac{5}{7}$.

First $11\frac{1}{3}=\frac{\overline{11\times3}+1}{3}=\frac{34}{3}$.

And $3\frac{5}{7}=\frac{\overline{3\times7}+5}{7}=\frac{26}{7}$.

Then $\frac{34\times26}{3\times7}=\frac{884}{21}=42\frac{2}{21}$, the required Product.

EXAMPLE X.

Multiply 17*l.* 15*s.* by 17*l.* 15*s.*

Thus $17\frac{3}{4}$ by $17\frac{3}{4}$.

Then $17\frac{3}{4}=\frac{\overline{17\times4}+3}{4}=\frac{71}{4}$.

 Therefore

Therefore $\frac{71 \times 71}{4 \times 4} = \frac{5041}{16} = 315\frac{1}{16}$ the required Product.

EXAMPLE XI.

Multiply $584\frac{117}{524}$ by $81\frac{11}{17}$.

First $584\frac{117}{524} = \frac{\overline{584 \times 524} + 117}{524} = \frac{306133}{524}$.

And $81\frac{11}{17} = \frac{\overline{81 \times 17} + 11}{17} = \frac{1388}{17}$.

Therefore $\frac{306133}{524} \times \frac{1388}{17} = \frac{424912604}{8908} = 47700\frac{1004}{8908}$ the required Product.

CASE VI. EXAMPLE XII.

Multiply $17\frac{3}{7}$ by $\frac{5}{9}$.

Thus $17\frac{3}{7} = \frac{\overline{17 \times 7} + 3}{7} = \frac{122}{7}$.

And $\frac{122}{7} \times \frac{5}{9} = \frac{610}{63} = 9\frac{43}{63}$, the required Product.

EXAMPLE XIII.

Multiply $5\frac{3}{5}$ by $\frac{4}{7}$.

Thus $5\frac{3}{5} = \frac{\overline{5 \times 5} + 3}{5} = \frac{28}{5}$.

And $\frac{28}{5} \times \frac{4}{7} = \frac{112}{35} = 3\frac{1}{5}$ the required Product.

CASE VII. EXAMPLE XIV.

Multiply $\frac{21}{63}$ by 37.

Thus $\frac{37}{1} \times \frac{21}{63} = \frac{777}{63} = 12\frac{1}{3}$, the required Product.

EXAMPLE XV.

Multiply $\frac{5}{7}$ by 39.

Thus $\frac{5}{7} \times \frac{39}{1} = \frac{195}{7} = 27\frac{6}{7}$, the required Product.

This being the ſame as Caſe 3. I ſhall not inſiſt any more on it, ſeeing ſuch Operations are evidently eaſy by Inſpection, and therefore ſhall proceed to

CASE

CASE VIII. EXAMPLE XVI.

Multiply $\frac{3}{7}$ by $11\frac{3}{5}$.

Thus $11\frac{3}{4} = \frac{\overline{11\times5+3}}{5} = \frac{58}{5}$.

And $\frac{58}{5}\times\frac{3}{7} = \frac{174}{35} = 4\frac{34}{35}$, the required Product.

EXAMPLE XVII.

Multiply $\frac{11}{13}$ by $17\frac{13}{27}$.

Thus $17\frac{13}{27} = \frac{\overline{17\times27+13}}{27} = \frac{472}{27}$.

And $\frac{472}{27}\times\frac{11}{13} = \frac{5192}{351} = 14\frac{278}{351}$, the required Product.

This being the ſame as Caſe 5. ſhall not trouble the Reader with any more in this Caſe.

CASE IX. EXAMPLE XVIII.

Multiply $\frac{2}{3}$ of $\frac{3}{4}$ by $\frac{5}{6}$ of $\frac{6}{7}$.

Thus $\frac{2}{3}$ of $\frac{3}{4} = \frac{6}{12} = \frac{1}{2}$.

And $\frac{5}{6}$ of $\frac{6}{7} = \frac{30}{42} = \frac{5}{7}$.

Then $\frac{6}{12}\times\frac{30}{42} = \frac{180}{504} = \frac{5}{14}$.

So $\frac{1}{2}\times\frac{5}{7} = \frac{5}{14}$, the required Product.

CASE X. EXAMPLE XIX.

Multiply $\frac{3}{5}$ and $\frac{5}{9}$ by $\frac{2}{3}$ and $\frac{6}{11}$.

Thus $\frac{3}{5}$ and $\frac{5}{9} = \frac{25+27}{45} = \frac{52}{45}$.

And $\frac{2}{3}$ and $\frac{6}{11} = \frac{18+22}{33} = \frac{40}{33}$.

Then $\frac{52}{45}\times\frac{40}{33} = \frac{2080}{1485} = 1\frac{119}{297}$, the required Product.

A QUESTION.

What Number is $\frac{2}{7}$ of 21.

Thus $\frac{21}{1}\times\frac{2}{7} = \frac{42}{7} = 6$.

For $1 : \frac{2}{7} :: 21 : 6$.

Or $7 : 2 :: 21 : 6$.

Here, you ſee, is no Difficulty in managing Vulgar Fractions in Multiplication, if ſo that you obſerve the Method of reducing them as was done in *Addition* and *Subtraction*.

DIVISION.

PROPOSITION XVIII.

TO divide a Fraction by another, as $\frac{4}{5}$ by $\frac{2}{3}$.

Invert the Denominator, that is, make it $\frac{3}{2}$, then multiply the Dividend by it, and it's $\frac{12}{10}$ which is the Divisor.

DEMONSTRATION.

Because the Divisor is to the Dividend, as Unity to the Quotient, then the Dividend will be to the Divisor, as the Quotient to Unity; and if the Fractions be brought to the same Denominator, seeing they are equal to the Quotients from the Division of the Numerator by the Denominator, the Numerator of the divided Fraction will be to that of the dividing, as the Fraction divided to the one dividing, and so the Numerator of the Dividend is to that of the Divisor, as the Quotient to Unity; wherefore the Fractions being brought to one Denominator, the Numerator of the Dividend is to be divided by the Numerator of the Divisor to obtain the Quotient, and it will be $\frac{12}{10}$. Therefore for the Division of Common Fractions, this is the

RULE.

Multiply the Dividend by the Divisor's reverse Fraction, or which is the same, imagine the Terms of the Divisor changed, then work as in Multiplication.

EXAMPLE.

Thus $\frac{2}{3})\frac{4}{5}(= \frac{12}{10} = \frac{6}{5} = 1\frac{1}{5}$.

That is $\frac{3}{2} \times \frac{4}{5} = \frac{12}{10}$.
For $1 : \frac{3}{2} :: \frac{4}{5} : \frac{12}{10}$.
But $\frac{2}{3} : 1 :: 1 : \frac{3}{2}$ by *Cor.* 4. *pag.* 14.
And $\frac{2}{3} : 1 :: \frac{4}{5} : \frac{12}{10}$.

There-

Therefore $\frac{4}{5} \div \frac{2}{3} = \frac{12}{10}$, by the Nature of Proportion.

If either *Dividend* or *Divisor* be *whole* or *mixt* Numbers, or if both be mixt Numbers, *reduce them to improper Fractions.*

If they are Compound Fractions, *reduce them to single ones.*

If the *Fractions* are of one *Denomination*,

Then cast off that Denominator, and divide the *Numerators.*

Because *Fractions* having the same *Denominators*, are as their Numerators. *Cor.* 3. *pag.* 14.

S C H O L I U M.

The *Quotient* of any *Quantity* divided by a *proper Fraction* is always *greater* than that *Quantity.*

For in *dividing* by *Unit*, the *Quotient* will be equal to the *Dividend.*

But a *less Divisor* gives a *greater Quotient.*

Therefore in *dividing* by a *proper Fraction*, (that is, by less than an Unit,) the Quotient must be greater than the Dividend.

Or thus $\frac{2}{3})\frac{4}{5}(= \frac{12}{10} = \frac{6}{5}$, and $\frac{6}{5}$ is greater than $\frac{4}{5}$.

For $1 : \frac{6}{5} :: \frac{2}{3} : \frac{4}{5}$, by the Nature of Division.

And $1 : \frac{2}{3} :: \frac{6}{5} : \frac{4}{5}$ by *Alternation.*

But $1 > \frac{2}{3}$, therefore $\frac{6}{5} > \frac{4}{5}$.

Note, $>$ *and* $<$ *are signs signifying greater and lesser.*

CASE I. EXAMPLE I.

Divide $\frac{11}{12}$ by $\frac{2}{3}$.

First invert the Divisor $\frac{2}{3}$ and it will be $\frac{3}{2}$.

Then $\frac{3}{2})\frac{11}{12}(= \frac{33}{24} = 1\frac{3}{8}$ the Quotient required.

EXAMPLE II.

Divide $\frac{3}{5}$ by $\frac{4}{7}$.

Thus $\frac{7}{4})\frac{3}{5}(= \frac{21}{20} = 1\frac{1}{20}$, the Quotient required.

CASE II. EXAMPLE III.

Divide $43\frac{1}{5}$ by 12.

Thus $43\frac{1}{5}=\frac{\overline{43\times5+1}}{5}=\frac{216}{5}$; then $\frac{12}{1}$ inverted,

And $\frac{1}{12})\frac{216}{5}(=\frac{216}{60}=3\frac{3}{5}$, the required Quotient, as is evident from *Case* 2. *Ex.* 3. in Multiplication.

EXAMPLE IV.

Divide $9660\frac{21}{71}$ by 171.

Thus $9660\frac{21}{71}=\frac{\overline{9660\times71+21}}{71}=\frac{685881}{71}$.

And 171 order'd $=\frac{171}{1}$.

Then $\frac{1}{171})\frac{685881}{71}(=57\frac{35}{71}$, the required Quotient, as appears from Multiplication.

CASE III. EXAMPLE V.

Divide $7\frac{6}{7}$ by $\frac{5}{7}$.

Thus $7\frac{6}{7}=\frac{\overline{7\times7+8}}{7}=\frac{55}{7}$.

And $\frac{7}{5})\frac{55}{7}(=\frac{385}{35}=11$, the Quotient required. See *Multiplication*, *Case* 3.

EXAMPLE VI.

Divide $139\frac{93}{121}$ by $\frac{112}{121}$.

Thus $139\frac{93}{121}=\frac{\overline{139\times121+93}}{121}=\frac{\overline{16819+93}}{121}=\frac{16912}{121}$.

And $\frac{121}{112})\frac{16912}{121}(=151$, the Quotient required.

CASE IV. EXAMPLE VII.

Divide $825\frac{1}{9}$ by 47.

Thus $825\frac{1}{9}=\frac{\overline{825\times9+1}}{9}=\frac{7426}{9}$.

Then $\frac{1}{47})\frac{7426}{9}(=\frac{7426}{423}=17\frac{5}{9}$, the required Quotient.

Ex-

EXAMPLE VIII.

Divide $16\frac{3}{11}$ by 4.

Thus $16\frac{3}{11}=\frac{\overline{16\times 11}+3}{11}=\frac{176+3}{11}=\frac{179}{11}$.

Then $\frac{1}{4})\frac{179}{11}(=\frac{179}{44}=4\frac{3}{44}$, the required Quotient.

CASE V. EXAMPLE IX.

Divide $42\frac{2}{21}$ by $3\frac{5}{7}$.

Thus $42\frac{2}{21}=\frac{\overline{42\times 21}+2}{21}=\frac{882+2}{21}=\frac{884}{21}$.

And $3\frac{5}{7}=\frac{\overline{3\times 7}+5}{7}=\frac{21+5}{7}=\frac{26}{7}$.

Then $\frac{7}{26})\frac{884}{21}(=\frac{6188}{546}=11\frac{1}{3}$, the required Quotient.

EXAMPLE X.

Divide $19\frac{13}{17}$ by $7\frac{23}{31}$.

Thus $19\frac{13}{17}=\frac{\overline{19\times 17}+13}{17}=\frac{323+13}{17}=\frac{336}{17}$.

And $7\frac{23}{31}=\frac{\overline{7\times 31}+23}{31}=\frac{217+23}{31}=\frac{240}{31}$.

Then $\frac{31}{240})\frac{336}{17}(=\frac{10416}{4080}=2\frac{47}{85}$, the required Quotient.

CASE VI. EXAMPLE XI.

Divide $17\frac{3}{5}$ by $\frac{5}{9}$.

Thus $17\frac{3}{5}=\frac{\overline{17\times 5}+3}{5}=\frac{85+3}{5}=\frac{88}{5}$.

Then $\frac{9}{5})\frac{88}{5}(=\frac{792}{25}=31\frac{17}{25}$, the Quotient required.

EXAMPLE XII.

Divide $29\frac{11}{13}$ by $\frac{13}{29}$.

Thus $29\frac{11}{13}=\frac{\overline{29\times 13}+11}{13}=\frac{377+11}{13}=\frac{388}{13}$.

Then $\frac{29}{13})\frac{388}{13}(=\frac{11252}{169}=66\frac{98}{169}$, the required Quotient.

CASE

CASE VII. EXAMPLE XIII.

Divide $\frac{21}{31}$ by 37.

Thus $\frac{1}{37})\frac{21}{31}($ $=\frac{21}{1147}$, the required Quotient.

EXAMPLE XIV.

Divide $\frac{3}{11}$ by 19.

Thus $\frac{1}{19})\frac{3}{11}($ $=\frac{3}{209}$, the required Quotient.

CASE VIII. EXAMPLE XV.

Divide $\frac{3}{5}$ by $17\frac{1}{2}$.

Thus $17\frac{1}{2}=\frac{\overline{17\times2}+1}{2}=\frac{35}{2}$.

Then $\frac{2}{35})\frac{3}{5}($ $=\frac{6}{175}$, the required Quotient.

CASE IX. EXAMPLE XVI.

Divide $\frac{3}{5}$ of $\frac{5}{7}$ by $\frac{5}{9}$.

Thus $\frac{3}{5}$ of $\frac{5}{7}=\frac{15}{35}=\frac{3}{7}$.

And $\frac{9}{5})\frac{3}{7}($ $=\frac{27}{35}$, the required Quotient.

EXAMPLE XVII.

Divide $\frac{3}{7}$ of $\frac{4}{11}$ and $\frac{3}{5}$ by $\frac{2}{3}$.

Thus $\frac{3}{7}$ of $\frac{4}{11}=\frac{12}{77}$.

And $\frac{12}{77}+\frac{3}{5}=\frac{60+231}{385}=\frac{291}{385}$.

Then $\frac{3}{2})\frac{291}{385}($ $=\frac{873}{770}=1\frac{103}{770}$, the Quotient, and so any compound Fraction.

A QUESTION.

What Number does 6 contain $\frac{2}{7}$ of?

Thus $\frac{2}{7}$ inverted is $\frac{7}{2}$ and $\frac{7}{2})\frac{6}{1}($ $\frac{42}{2}=21$.

For $\frac{2}{7}:1::6:21$.

Or $2:7::6:21$, Answer.

By what has been done, may be perform'd all manner of *Additions*, *Subtractions*, *Multiplications* and *Divisions*

ſions in Fractions. The Reaſon why I have taken the Numbers ſo ſmall in the foregoing Examples, was on purpoſe, that I might not perplex the Mind of the Learner, ſince little familiar Examples are ſufficient, and more proper to enlighten the Underſtanding than larger ones.

Methinks it would not be improper, if we come now to put the preceding Rules into Uſe, and explain a little the Nature of Fractions in anſwering Queſtions.

But before I begin with Queſtions relating to Proportion, as in the Rule of Three Direct, &c. it will be requiſite to premiſe the following Propoſition, which will be found uſeful in Queſtions of other Kinds.

PROPOSITION XIX.

WHEN Number is applied to any Quantity, then that Quantity doth, or is imagined, to contain ſo many equal Quantities of the ſame Kind, as the Number applied doth contain Units; ſo that of Quantities of one Kind compared together, when the one is ſaid to be one Number, the other another Number, the expreſſing and naming of the *Ratio* which is between them, is nothing but to find a Number, which ſhall have ſuch *Ratio* to 1, as of the two Numbers applied the greater hath to the leſſer, which in conſideration that the Compariſon required is to 1, if of the Numbers applied you divide the greater by the leſſer, the Quotient will be the Number you deſire, expreſſing the *Ratio* demanded; and is named by ſaying the Number found, adding this Word *times*, if the Quantities compared be in the greater Inequality; but if in the leſſer with this Word *under* put before, to make Diſtinction of the one and the other.

EXAMPLE.

3 to 2 is $1\frac{1}{2}$ *times*.
For $\frac{3}{2} = 1\frac{1}{2}$.
Alſo 5 to 3 is $1\frac{2}{3}$ *times*.
For $\frac{5}{3} = 1\frac{2}{3}$.

And

And 15 to 4 is $3\frac{3}{4}$ times.
For $\frac{15}{4} = 3\frac{3}{4}$.
But 2 to 3 is *under* $1\frac{1}{2}$ *times.*
And 3 to 5 is *under* $1\frac{2}{3}$ *times.*
Also 3 to 7 is *under* $2\frac{1}{3}$ *times.*
And 4 to 15 is *under* $3\frac{3}{4}$ *times.*
Moreover 2 to 1 is 2 *times.*
6 to 2 is 3 *times.*
12 to 3 is 4 *times.*
But 1 to 2 is *under* 2 *times.*
2 to 6 is *under* 3 *times.*
3 to 12 is *under* 4 *times*, &c. *in infinitum.*

Ever in all Things effectually respecting what is intended to be expressed, or signified by the written Figures, Points, Pricks, Lines, *&c.* As in this place the intent is to shew, that of two Quantities, or Numbers of one Kind compared in the greater Inequality, the Antecedent doth contain the Consequent, as the *Ratio* doth express; and in the lesser Inequality, that the Antecedent is contained in the Consequent, as the Ratio doth signify. Likewise again, when 4 Quantities given of one Kind are proportional, as the Ratio between the first and the second, is one and the same that is between the third and the fourth; so the *Ratio* between the first and the third, of the same Quantities so given is one and the same that is between the second and the fourth. Further, that that *Ratio* which is between the Quantities of one Kind compared together, the same *Ratio* is between the Part or Parts of the same Antecedent, and the like Part or Parts of the same Consequent.

The

PROPOSITION XX.

The RULE OF THREE Direct.

PROBLEM.

THREE Numbers being given to find a fourth.
The Rule is the same as in whole Numbers.

EXAMPLE I.

If the $\frac{1}{8}$ of a Yard cost $\frac{1}{6}$*l.* what will $\frac{1}{3}$ Yard cost.

Thus, if $\frac{1}{8} : \frac{1}{6} :: \frac{1}{3} : \frac{4}{9}$.
For $\frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$,
And $\frac{8}{1})\frac{1}{18}(\frac{8}{18} = \frac{4}{9} = 8$*s.* 10*d.* $\frac{1}{2}\,\frac{2}{3}$.

EXAMPLE II.

If $\frac{2}{9}$*l.* buy $\frac{5}{8}$ Yard, what will $\frac{4}{7}$*l.* buy?

Thus $\frac{2}{9} : \frac{5}{8} : \frac{4}{7} : 1\frac{17}{28}$.
For $\frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$.
And $\frac{9}{2})\frac{20}{56}(\frac{180}{112} = 1\frac{17}{28}$ Yards.

EXAMPLE III.

If $\frac{5}{9} : \frac{3}{7} : \frac{3}{11} : \frac{81}{385}$.
For $\frac{3}{7} \times \frac{3}{11} = \frac{9}{77}$.
And $\frac{9}{5})\frac{9}{77}(\frac{81}{385}$.

EXAMPLE IV.

If $\frac{9}{13}$ of Hogshead of Wine cost 31*l.* $\frac{3}{7}$, what will $\frac{5}{7}$ Hogshead cost?

If $\frac{9}{13} : 31\frac{3}{7} :: \frac{5}{7} : 32\frac{188}{441}$.

For $31\frac{3}{7} = \frac{31 \times 7 + 3}{7} = \frac{217 + 3}{7} = \frac{220}{7}$.

And $\frac{220}{7} \times \frac{5}{7} = \frac{1100}{49}$.

Then $\frac{13}{9})\frac{1100}{49}(= \frac{14300}{441} = 32\frac{188}{441} = 32$*l.* 8*s.* 6*d.* $\frac{1}{4}\,\frac{111}{441}$

Ex

EXAMPLE V.

If $\frac{2}{3}$ of $\frac{3}{4}$ require 5, what will $\frac{4}{5}$ of $\frac{11}{12}$ require.

If $\frac{2}{3}$ of $\frac{3}{4} : 5 :: \frac{4}{5}$ of $\frac{11}{12} : 7\frac{1}{3}$.
Thus $\frac{2}{3}$ of $\frac{3}{4} = \frac{6}{12} = \frac{1}{2}$.
And $\frac{4}{5}$ of $\frac{11}{12} = \frac{44}{60} = \frac{11}{15}$.
Then $\frac{11}{15} \times \frac{5}{1} = \frac{55}{15}$.
And $\frac{2}{1})\frac{55}{15}(\frac{110}{15} = 7\frac{1}{3}$, Anſwer.

EXAMPLE VI.

If $\frac{3}{5}$ of $\frac{2}{3}$ and $\frac{5}{9}$ require $\frac{8}{11}$ of $\frac{7}{16}$ of $\frac{5}{9}$ and $\frac{3}{4}$, what will 25 require?

If $\frac{3}{5}$ of $\frac{2}{3}$ and $\frac{5}{9} : \frac{8}{11}$ of $\frac{7}{16}$ of $\frac{5}{9}$ and $\frac{3}{4} :: 25 : 29\frac{21626}{34056}$.
Firſt $\frac{3}{5}$ of $\frac{2}{3} = \frac{6}{15} = \frac{2}{5}$.

And $\frac{2}{5} + \frac{5}{9} = \dfrac{25+18}{45} = \frac{43}{45}$.

Then alſo $\frac{8}{11}$ of $\frac{7}{16}$ of $\frac{5}{9} = \frac{280}{1584} = \frac{35}{191}$.

And $\frac{35}{198} + \frac{3}{4} = \dfrac{594+140}{792} = \frac{734}{792}$.

Now the Terms being order'd according to the preceding Rules when ſtated will ſtand thus:

$\frac{43}{45} : \frac{734}{792} :: \frac{25}{1} : 29\frac{21626}{34056}$.
For $\frac{734}{792} \times \frac{25}{1} = \frac{18350}{792}$.
And $\frac{55}{43})\frac{18350}{792}(= \frac{1009250}{34056} = 29\frac{21626}{34056}$.

EXAMPLE VII.

If $\frac{1}{3}$ and $\frac{3}{4}$ of $\frac{3}{5}$ require $23\frac{1}{3}$, what will $\frac{1}{7}$ of $\frac{8}{9}$ and $\frac{2}{3}$ require?

Thus $\frac{3}{4}$ of $\frac{3}{5} = \frac{9}{20}$.

And $\frac{1}{3} + \frac{9}{20} = \dfrac{27+20}{60} = \frac{47}{60}$.

Alſo $23\frac{1}{3} = \dfrac{23\times3+1}{3} = \dfrac{69+1}{3} = \frac{70}{3}$.

Alſo $\frac{1}{7}$ of $\frac{8}{9} = \frac{8}{63}$.

And $\frac{8}{63} + \frac{2}{3} = \dfrac{24+126}{189} = \frac{150}{189}$.

Then

Then $\frac{47}{60} : \frac{70}{3} :: \frac{150}{189} : 23\frac{17073}{26649}$.

$\frac{150}{189} \times \frac{70}{3} = \frac{10500}{567}$.

And $\frac{60}{47}) \frac{10500}{567} (= \frac{630000}{26649} = 23\frac{17073}{26649}$.

There is another Method whereby the Rule of Three Direct may be performed in Vulgar Fractions, and in my Opinion is much neater, which is thus performed by observing this Rule.

RULE.

The Numerator of the first Term multiplied into the Denominator of the second and third, the Product is a new Denominator; and the Denominator of the first Term multiplied into the Numerator of the second and third, the Product is a new Numerator; which new Fraction is the Answer, it being a fourth Proportional.

An Example or two will render this familiar.

EXAMPLE I.

If $\frac{1}{8}$ Yard cost $\frac{1}{6}$ *l.* what will the $\frac{1}{3}$ of a Yard cost.

1st. 2d. 3d. 4th.

Thus $\frac{1}{8} : \frac{1}{6} :: \frac{1}{3} : \frac{8}{18} = \frac{4}{9}$.

$$\left.\begin{array}{l}\text{For } 1\times6\times3 = 18 \text{ a Denominator}\\ \text{And } 8\times1\times1 = 8 \text{ a Numerator}\end{array}\right\} = \frac{8}{18} \text{ or } \frac{4}{9}.$$

EXAMPLE II.

If $\frac{2}{9}$ *l.* buy $\frac{5}{8}$ Yard, what will $\frac{4}{7}$?

Thus $\frac{2}{9} : \frac{5}{8} :: \frac{4}{7} : 1\frac{17}{28}$.

$$\left.\begin{array}{l}\text{For } 2\times8\times7 = 112 \text{ Denominator}\\ \text{And } 9\times5\times4 = 180 \text{ Numerator}\end{array}\right\} = \frac{180}{112} = 1\frac{17}{28}.$$

EXAMPLE III.

If $\frac{5}{9} : \frac{3}{7} :: \frac{3}{11} : \frac{81}{385}$.

For $5\times7\times11 = 385$ Denominator.

And $9\times3\times3 = 81$ Numerator.

Then $\frac{81}{385}$ is the Answer, or 4th Proportional.

As is manifest from the 1st, 2d, and 3d. Examples of this Proposition.

A Collection of Questions promiscuously set, exercising the preceding Rules.

QUESTION I.

What is the $\frac{3}{8}$ of $\frac{1}{2}$ a Mark?

$$\text{Thus } \tfrac{1}{2} \text{ a Mark} = \overset{s.}{6}\ \overset{d.}{8}. \qquad \text{Then} \quad \begin{array}{r} \overset{s.}{6}\ \ \overset{d.}{8} \\ 3 \\ \hline 8)1\ \ 0\ \ 0 \\ \hline 2\ \ 6 \end{array} \text{ Answer.}$$

QUESTION II.

What is the $\frac{5}{11}$ of $13d.\ \frac{1}{2}$?

$$\text{Thus} \quad \begin{array}{r} \overset{s.}{1}\ \ \overset{d.}{1\tfrac{1}{2}} \\ 5 \\ \hline 11)5\ \ 7\tfrac{1}{2} \\ \hline 6\tfrac{6}{11} \end{array}$$

QUESTION III.

What Quantity is that from which if I take $3\frac{2}{3}$, the Remainder shall be $2\frac{1}{5}$?

$$\text{Thus } 3\tfrac{2}{3} = \frac{3\times3+2}{3} = \frac{9+2}{3} = \tfrac{11}{3}.$$

$$\text{And } 2\tfrac{1}{5} = \frac{2\times5+1}{5} = \frac{10+1}{5} = \tfrac{11}{5}.$$

$$\text{Then } \tfrac{11}{3} + \tfrac{11}{5} = \frac{33+55}{15} = \tfrac{88}{15} = 5\tfrac{13}{15} \text{ the Answer.}$$

PROOF.

For from $5\frac{13}{15} = \frac{88}{15}$ take $3\frac{2}{3} = \frac{11}{3}$.

$$\text{Thus } \tfrac{88}{15} - \tfrac{11}{3} = \frac{264-165}{45} = \tfrac{99}{45} = 2\tfrac{9}{45} = 2\tfrac{1}{5}.$$

QUE-

QUESTION IV.

What Quantity is that, to which if I add $3\frac{5}{7}$, the Sum shall be $5\frac{4}{35}$?

Thus from $5\frac{4}{35}=\frac{\overline{5\times35}+4}{35}=\frac{175+4}{35}=\frac{179}{35}$,

Take $3\frac{5}{7}=\frac{\overline{3\times7}+5}{7}=\frac{21+5}{7}=\frac{26}{7}$.

That is $\frac{179}{35}-\frac{26}{7}=\frac{1253-910}{245}=\frac{343}{245}=1\frac{2}{5}$ Answer.

QUESTION V.

A Merchant has $\frac{3}{16}$ and $\frac{1}{2}$ of $\frac{1}{4}$ of a Ship; what Part is that of the Whole?

Thus $\frac{1}{2}$ of $\frac{1}{4}=\frac{1}{8}$, and $\frac{3}{16}+\frac{1}{8}=\frac{16+24}{128}=\frac{40}{128}=\frac{5}{16}$.

QUESTION VI.

A Merchant has $\frac{1}{5}$ of $\frac{1}{8}$, and $\frac{5}{11}$ of $\frac{2}{3}$ of the Share of the Cargo of a Ship; what Part is that of the Whole?

Thus $\frac{1}{5}$ of $\frac{1}{8}=\frac{1}{40}$, and $\frac{5}{11}$ of $\frac{2}{3}=\frac{10}{33}$.

Then $\frac{1}{40}+\frac{10}{33}=\frac{400+33}{1320}=\frac{433}{1320}$.

QUESTION VII.

What is the Difference betwixt the Sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and an Unit?

Thus $2\times3\times4=24$ Denominator.
And $1\times3\times4=12$ Numerator.
$1\times2\times4=\ 8$ a Numerator.
$1\times2\times3=\ 6$ a Numerator.

That is $\frac{12+8+6}{24}=\frac{26}{24}=1\frac{1}{12}$.

Then $1\frac{1}{12}-1=\frac{1}{12}$ the Answer.

QUESTION VIII.

A certain Merchant having $\frac{3}{5}$ of a Ship, sells $\frac{3}{4}$ of his Share for 240*l*; what is the whole Ship worth?

Thus $\frac{3}{4}$ of $\frac{3}{5} = \frac{9}{20}$. Then 9 : 20 :: 240 : 533 $\frac{1}{3}$.
Answer 533*l*. $\frac{1}{3}$.

QUESTION IX.

A Father dying left his Son a certain Portion, of which he spent $\frac{1}{4}$, of the Rest he spent $\frac{1}{2}$, and then he had 800*l*; what was the Portion?

Thus $1 - \frac{1}{4} = \frac{3}{4}$, and $\frac{1}{2}$ of $\frac{3}{4} = \frac{3}{8}$; then $\frac{3}{4} - \frac{3}{8} = \frac{3}{8}$.
Then 3 : 8 :: 800 : 2133$\frac{1}{3}$.
Answer 2133*l*. $\frac{1}{3}$.

QUESTION X.

A younger Brother received 410*l*. which was $\frac{3}{4}$ of $\frac{2}{3}$ of his elder Brother's Portion; now 3$\frac{1}{2}$ times his elder Brother's Portion was 1$\frac{1}{2}$ of his Father's Estate; what was the Father's Estate?

Thus $\frac{3}{4}$ of $\frac{2}{3} = \frac{6}{12} = \frac{1}{2}$. Then as 1 : 2 :: 410 : 820, that is, as the Numerator of the Fraction is to its Denominator, so is 410*l*. to the elder Brother's Portion = 820.

Then $3\frac{1}{2} = \frac{3 \times 2 + 1}{2} = \frac{7}{2}$, and $\frac{820}{1} \times \frac{7}{2} = \frac{5740}{2} = 2870$ $= 1\frac{1}{3}$ (or $\frac{4}{3}$) of the Father's Estate.

That is, $\frac{820}{1} \times \frac{7}{2} = \frac{5740}{2} = 2870$; now 1$\frac{1}{2}$ times 2870 = 4305*l*. the Father's Estate.

Seeing the elder Brother's Portion = 820, and as it is likewise 1$\frac{1}{3}$ of the Father's Estate, it will be

4 : 3 :: 3$\frac{1}{2}$ Brother's Portion : Father's Estate.

QUESTION XI.

What Part of 3*d*. is $\frac{2}{3}$ of 2*d*?
Thus 2 : $\frac{2}{3}$ :: 3 : $\frac{4}{9}$.
For $\frac{2}{3} \times \frac{2}{1} = \frac{4}{3}$, and $\frac{1}{3}$)$\frac{4}{3}$($= \frac{4}{9}$ Answer.

QUESTION XII.

What Money is that in Proportion to 20*s.* as 5 is to $13\frac{1}{2}$?

Thus $13\frac{1}{2} : 20 :: 5 : 7\frac{11}{27}$.

For $20 \times 5 = 100$, and $13\frac{1}{2} = \frac{\overline{13 \times 2} + 1}{2} = \frac{27}{2}$.

Then $\frac{2}{27})\frac{100}{1}(\frac{200}{27} = 7\frac{11}{27}$ Answer,
Or 7*s.* 4*d.* $\frac{3}{4}$ $\frac{5}{9}$.

$20 : 185 : 100$
$a \quad b \quad c$

$100 : 20 \; 185$

$\frac{bc}{a}$

$\frac{bc}{a} = x \qquad x + a = \frac{b+c}{a} - x$

OF
DECIMAL FRACTIONS.

DEFINITION.

AN Unit *may be imagined to be divided equally into* 10 *Parts, and each of those into* 10 *more; so that, by a continual Decimal Subdivision, the* Unit *may be supposed to be divided into* 10, 100, 1000, &c. *equal Parts, called* 10*th*, 100*th*, 1000*th Parts of an* Unit.

And as Integers increase from Unit, towards the left Hand, in a tenfold Proportion, so that a Figure in any Place is *ten* times as many in the next Place below it, and but a *tenth* Part of what it signifies in the next Place above it; therefore as the 1st, 2d, 3d, *&c. Place* above that of Units, is *Tens*, *Hundreds*, *Thousands*, &c. So the 1st, 2d, 3d, *&c. Place* below that of Units, is *Tenths*, *Hundredths*, *Thousandths*, &c. decreasing in a subdecuple Proportion; as is evident from the following Table.

	Integers							Parts						
	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths	Tens of Thousandths	Hundred Thousandths	Millionths	
&c.	6	5	4	3	2	1	0	1	2	3	4	5	6	&c.

Now

Now because the Denominators of Decimal Fractions differ only in the Number of Places, and not in the Figures, they being always an Unit with Cyphers, they may be expressed without their Denominators, with a Point prefixed; thus $\frac{6}{10}$ is expressed decimally .6, and $\frac{35}{100}$ is decimally expres'd .35, also $\frac{254}{1000}$ is expres'd thus .254, where observe, this dot (.) distinguishes them from whole Numbers.

Thus	.5	Signifies	5 *Tenths,*
	.25		25 *Hundredths,*
	.175		175 *Thousandths.*

COROLLARY.

As Cyphers set on the right Hand of Integers do increase the Value of them decimally, as 5, 50, 500, *&c.* when set on the left Hand of Decimal Fractions, they decrease the Value decimally, as .5, .05, .005, *&c.* But set on the left Hand of Integers, or on the right Hand of Decimal Fractions, they signify nothing, but only to fill up void Places. Thus .50000 is but 5 Tenths, and 00005. is but 5 Units.

SCHOLIUM I.

Hence 1, 2, 3, *&c.* Cyphers, before a Decimal, advance it so many Places forward, whereby it is made 10, 100, 1000, *&c.* times less. Thus,

.25	Signifies	25 *Hundredths,*
.025		25 *Thousandths,*
.0025		25 *Ten Thousandths,*
.00025		25 *Hundred Thousandths.*

SCHOLIUM II.

Therefore, a Figure in the 1st, 2d, 3d, *&c. decimal Place*, is 10, 100, 1000, *&c.* times less than if it were an Integer.

SCHOLIUM III.

Consequently, each Removal of a *Figure* into a *Place*

forward.

forward makes it *ten* Times leſs than it was before. For .05 is five hundredth Parts of an Integer, but .50 is no more than five tenth Parts of an Integer. Likewiſe .500, or .5000, .50000 are all the ſame Value, namely .5.

I have been as clear as poſſible in explaining the Notation of *theſe* Decimals, becauſe of their vaſt Extenſiveneſs, and the great Facility they bring with their Practice in ſeveral Operations, not only in Arithmetic, but in moſt other Parts of the Mathematics.

For arithmetical Operations may be perform'd vaſtly ſooner by Decimal Fractions than by Vulgar Fractions, becauſe the Denominators being omitted, the Rules of Addition, Subtraction, Multiplication and Diviſion are performed as in whole Numbers, Regard being had to the Pointing, which is very eaſy: Yet, I muſt ſay, by theſe, Operations will not always come out exactly true; but you may come as near the Truth as poſſible, by bringing out more Figures.

Regiomontanus was the firſt that uſed Decimal Fractions in the Conſtruction of the Tables of Sines, about *A. D.* 1464.

PROPOSITION I.

TO *reduce* Vulgar Fractions *into* Decimals.

RULE.

To the Numerator add as many Cyphers as you would have Decimal Places; then divide it by the Denominator, and the Quotient (if there be no Remainder) will be a Decimal equivalent to the Vulgar Fraction given.

But when there is a Remainder, you may, by adding more Cyphers, proceed ſo as to bring the Quotient nearly equal.

EXAMPLE I.

Thus $\frac{1}{4} = \frac{1.00}{4} = .25$, for $4 : 1 :: 1.00 : .25$

Ex-

EXAMPLE II.

$\frac{1}{2} = \frac{1.0}{2} = .5$, for $2 : 1 :: 1.0 : .5$.

EXAMPLE II.

$\frac{3}{4} = \frac{3.00}{4} = .75$, for $4 : 3 :: 1.00 : .75$.

EXAMPLE IV.

$\frac{2}{7} = \frac{2.000000 \&c.}{7} = .285714 = \frac{2}{7}$ nearly, not wanting $\frac{1}{1000000}$ *Part* of an *Unit*. For $7 : 2 :: 1.000000$ &c. $: .285714$ &c.

EXAMPLE V.

$\frac{15}{19} = \frac{15.00000 \&c.}{19} = .78947$ &c.

EXAMPLE VI.

$\frac{1}{32} = \frac{1.00000}{32} = .03125$.

Note, That becauſe I annex five Cyphers to 1 the given Numerator, (as I intended to have five Places of Decimals) and there ariſe but four Figures in the Quotient, I muſt ſupply ſuch Defect by prefixing as many Cyphers on the left Hand of the firſt Figure in the Quotient as there want Places; as in the preceding $\frac{1}{32}$, where the Quote conſiſted but of 4 Places, here I annex a Cypher on the left Hand of 3, the firſt Figure in the Quotient, and then it becomes .03125 the true Decimal required.

EXAMPLE VII.

Reduce $\frac{2}{3}$ of $\frac{3}{4}$ into a Decimal Fraction.

Thus $\frac{2}{3}$ of $\frac{3}{4} = \frac{6}{12} = \frac{1}{2}$, then $\frac{1.0}{2} = .5$.

EXAMPLE VIII.

Reduce $\frac{5}{7}$ and $\frac{3}{5}$ of $\frac{2}{7}$ into a Decimal Fraction.

Thus $\frac{3}{5}$ of $\frac{2}{7} = \frac{6}{35}$ and $\frac{5}{7} + \frac{6}{35} = \frac{42+175}{245} = \frac{217}{245}$.

Then $\frac{217.000000 \&c.}{245} = .885714$ &c.

And ſo any Compound Fraction may be reduced into a Decimal one, by reducing it into a ſingle one, according to the Rules of Reduction.

COR-

COROLLARY.

These being understood, it will be no difficult Thing to find the Decimal Parts, answerable to any known Part or Parts of *Coins*, *Weights*, *Measures*, &c. if you reduce first the given Parts of *Coins*, *Weights*, *Measures*, &c. into a Vulgar Fraction, whose *Denominator* is the *Number* of those known *Parts* contained in the *Integer*, and the given *Parts* its *Numerator*. An Example or two will, I hope, render it familiar.

EXAMPLE I.

What is the Decimal of 6*s.* 8*d*?

Thus 6*s.* 8*d.* = 80*d.* and 20*s.* = 240*d.* Then $\frac{80}{240} = \frac{1}{3}$ the required Vulgar Fraction; therefore $\frac{1.00000\ \&c.}{3} = \cdot 33333$ &c.

EXAMPLE II.

What is the Decimal of 15*s*?

Thus $\frac{15}{20} = \frac{3}{4}$, and $\frac{3.00}{4} = \cdot 75$.

EXAMPLE III.

What is the Decimal of 16*s.* 7*d* $\frac{1}{2}$?

Thus 16*s.* 7*d.* $\frac{1}{2}$ = 798 Farthings, and 960 Farthings = 1*l.*

Then $\frac{798}{960} = \frac{133}{160}$; therefore $\frac{133.0000\ \&c.}{160} = .8312$ &c.

EXAMPLE IV.

What is the Decimal of 19*s.* 11*d.* $\frac{3}{4}$?

It is evident at Sight that 19*s.* 11*d.* $\frac{3}{4}$ wants but *one* Farthing of a Pound (960,) and therefore it will be $\frac{959}{960}$ the Fraction; then $\frac{959.000000\ \&c.}{960} = \cdot 998957$ &c.

EXAMPLE V.

What is the Decimal of $\frac{3}{4}$*qr.* 26*lb.* Avoirdupois Weight, when the Integer is *one Hundred Weight*?

Thus

Thus 1 *C.* = 112 *lb.* and $\frac{3}{4}$ *qr.* + 26 *lb.* = 110 *lb.* then $\frac{110}{112}$ = $\frac{55}{56}$; Therefore $\frac{55.0000 \text{ \&c.}}{56}$ = .9821 &c. the required Decimal.

EXAMPLE VI.

What is the Decimal of 5 *C.* 1 *qr.* 11 *lb.* the Integer being a *Tun Weight?*

Thus 2240 *lb.* = a Tun Weight, and 5 *C.* 1 *qr.* 11 *lb.* = 599 *lb.* then $\frac{599}{2240}$ is the required Vulgar Fraction; and therefore $\frac{599.0000 \text{ \&c.}}{2240}$ = .2674 &c. the required Decimal.

EXAMPLE VII.

What is the Decimal of 7 *d.* $\frac{3}{4}$, *one Shilling* being Integer?

First 7 *d.* is $\frac{7}{12}$ of a Shilling, and $\frac{3}{4}$ of a Shilling is $\frac{3}{48}$.

But $\frac{7}{12} + \frac{3}{48} = \frac{336+36}{576} = \frac{372}{576} = \frac{31}{48}$.

Then $\frac{31.00000 \text{ \&c.}}{48}$ = .64583 &c. the required Decimal.

Note, If the Integer had been one foot instead of 7 *d.* $\frac{3}{4}$, it would be 7 $\frac{3}{4}$ foot, the Conclusion being the same.

EXAMPLE VIII.

What is the Decimal of 5 *oz.* 11 *pw.* 16 *gr.* the Integer being 1 *lb?*

Thus 5 *oz.* 11 *pw.* 16 *gr.* = 2680 *gr.* and 1 *lb.* = 5760 *gr.* then $\frac{2680}{5760} = \frac{67}{144}$.

And $\frac{67.00000 \text{ \&c.}}{144}$ = .46527 &c. the Decimal required.

SCHOLIUM.

From what has been said may Decimal Tables be made for any Number; but I shall not in this place trouble the Reader with them, but only shew how he may make any for his Use, by a familiar Example or two.

EXAMPLE IX.

What is the Decimal of 19 *s.* a Pound being Integer?

Here I consider that it is $\frac{19}{20}$, then $\frac{19.00}{20}$ = .95, the Decimal required.

Also

Alſo,

s.				s.
18 is	$\frac{18}{20} = \frac{18.00}{20} = .9$	hence the Decimal of	18	$= .9$
17	$\frac{17}{20} = \frac{17.00}{20} = .85$		17	$= .85$
16	$\frac{16}{20} = \frac{16.0}{20} = .8$		16	$= .8$
15	$\frac{15}{20} = \frac{15.00}{20} = .75$		15	$= .75$
14	$\frac{14}{20} = \frac{14.00}{20} = .7$		14	$= .7$
13	$\frac{13}{20} = \frac{13.00}{20} = .65$		13	$= .65$
12	$\frac{12}{20} = \frac{12.0}{20} = .6$		12	$= .6$
11	$\frac{11}{20} = \frac{11.00}{20} = .55$		11	$= .55$
10	$\frac{10}{20} = \frac{10.0}{20} = .5$		10	$= .5$
9	$\frac{9}{20} = \frac{9.00}{20} = .45$		9	$= .45$
8	$\frac{8}{20} = \frac{8.0}{20} = .4$		8	$= .4$
7	$\frac{7}{20} = \frac{7.00}{20} = .35$		7	$= .35$
6	$\frac{6}{20} = \frac{6.0}{20} = .3$		6	$= .3$
5	$\frac{5}{20} = \frac{5.00}{20} = .25$		5	$= .25$
4	$\frac{4}{20} = \frac{4.00}{20} = .2$		4	$= .2$
3	$\frac{3}{20} = \frac{3.00}{20} = .15$		3	$= .15$
2	$\frac{2}{20} = \frac{2.0}{20} = .1$		2	$= .1$
1	$\frac{1}{20} = \frac{1.00}{20} = .05$		1	$= .05$

And by this Method may any other Table be made.

SCHOLIUM.

'Tis obſerved when a Fraction is reduced to the ſmalleſt Terms; that if its *Denominator* be compounded only of the prime Numbers 2 and 5 (the Components of 10) the Decimal of that Fraction will be Determined.

But if the *Denominator* be compounded of any other *prime Numbers*, it will be Indetermined; and the ſame Figures will return again in Order, and continue to *circulate*, either by one Figure, or by two, three, *&c.* Figures, tho' never by more than the Number of *Units* in the *Denominator* leſs 1.

For the *Remainder* being always leſs than the *Diviſor*, therefore may be any Number leſs by 1 than it.

But in ſo many Operations, at moſt, as there are Units in the Diviſor, one of the Remainders muſt return again.

There-

Therefore, the ſame Figure in the *Quotient* muſt alſo return, and ſo continue the *Circulation.*

To find the Number of the circulating Figures.

The moſt accurate and incomparable Mr *Jones* gives theſe Obſervations.

1. Divide the Denominator by 2 and 5 as often as poſſible; if it come to be 99, 999, 9999, &c. or an *aliquot Part* of ſuch Number, or a Number compounded of 2 or 5, and ſuch aliquot Part; then the Number of the circulating Figures will be equal to ſo many Figures of 9, as there are in the Number found.

2. If one of the prime Numbers compounding the *Denominator* (excluding thoſe of 2 and 5) be not an *aliquot Part* of the other; then the Number of the circulating Figures will be equal to the Product of them required by thoſe compounding Prime Numbers.

3. And when the Denominator is compounded of 2, or 5, or any Power of them; then the circulating Figures begin at ſuch a Place of Decimals, as is denoted by the Index of 2 or 5, aſſumed in that compoſition, more 1.

PROPOSITION.

TO find the Value of a Decimal Fraction.

This is but converſe of the former, and therefore the Rule for finding the true Value of a Decimal is grounded upon the ſame Reaſon, as that for turning a Number into a Decimal.

For it will hold, as the Decimal Denominator is to its Numerator, ſo are the Parts of the next inferior Denominator to the Numerator or Number of ſuch Parts contained in the Decimal. And hence comes this

RULE.

Multiply your given Decimal by the Parts of the next inferior Denominator that is equal to the Integer, the Decimal gives the Part thereof; and if there be any Remainder, by the next inferior Denomination, &c. till all is done.

 This

This will be rendered very familiar by an Example or two.

EXAMPLE I.

What is the Value of .95 of a Pound?

Thus .95x20*s*.=19.00*s*. Answer 19*s*.

EXAMPLE II.

What is the Value of .125 of a Pound?

Thus .125x20=2.500, .500x12=6*d*. Answer 2*s*. 6*d*.

EXAMPLE III.

What is the Value of .8546 of a Pound?

Thus .8546x20=17.0920, .0920x12=1.104. So 17*s*. 1*d*. .104=.8546; and so of any other.

EXAMPLE IV.

What is the Value of .54361 of a Tun Weight?

Thus .54361x20=10.87220, .87220x4=3.48880 x28=13.68640, .6864x16=10.9824. Therefore .54361=10*C*. 3*qr*. 13*lb*. 10*dr*.

EXAMPLE V.

What is the Value of .9434 of an Ounce Troy?

Thus .9434x20=18.8680x24=20.832.
So .9434*oz*.=18*pw*. 20*gr*.

ADDI-

ADDITION.

PROPOSITION I.

To add Decimal Fractions.

RULE.

Whether the Numbers given be pure or mixed Decimals, or ſome of them whole Numbers, write them down under one another, in ſuch order, that the decimal Points on the left ſtand all in a line, or under one another; and the Figures all in diſtinct Columns, in order as they are removed from the Point either on the Right or Left: Then, beginning at the Column on the right Hand, add the Figures in every Column together, juſt as you do in whole Numbers, placing a Point in the Sum under the Points of the given Numbers.

EXAMPLES

	l.		
	54·95		
	37·05		
	36·8		
	21·75		
	37·5		
£.	188·05		
Or	188	1	0

l.		
3741·8196		
514·5		
91·76184		
9841·546		
51·95		
14241·57744		
14241	11	$6\frac{1}{2}$

	C.		
	541·		
	1·974		
	·568		
	·037		
	·009		
	543·588		
C.	543	3	9

	l.	*s.*	
	·371		
	·25		
	·8501		
	·384		
	58·		
	59·8551		
£.	59	17	1

SUBTRACTION.

PROPOSITION II.

To subtract Decimal Fractions.

THIS is to be done in the same way as Subtraction in whole Numbers, only observe that you place every Figure under that of the like Name as *Addition*, then subtract.

EXAMPLES.

From	584.164
Take	396.896
Remains	187.268

From 496 take .851

Thus	496.000
	.851
	495.149

From 851 take .9849

Thus	851.
	.9849
Remains	850.0151

From	681.01
Take	596.461
Remains	84.549

From	81.913
Take	79.04963
Remains	2.86337

From	11.1101
Take	9.999999
Remains	1.110101

PRO-

PROPOSITION III.

MULTIPLICATION

Of Decimal Fractions.

RULE.

MUltiply the Factors as if all were Integers; and the Decimals in the Product must be equal to the Sum of those in both Factors; if they are not, prefix Cyphers to supply the Defect.

For the Index of each Figure in the Product must be equal to the Sum of the Indices of the multiplied and multiplying Figures.

Thus, Mult. 3.52 by 4.3, the Product is 15.136.

For $\left\{\begin{array}{l} 3.52 = \frac{352}{100} \\ 4.3 = \frac{43}{10} \end{array}\right\}$ and $\frac{352}{100} \times \frac{43}{10} = \frac{15136}{1000} = 15.136.$

Also .013 *mult.* by .005, gives .000065.

For $\left\{\begin{array}{l} .013 = \frac{13}{1000} \\ .005 = \frac{5}{1000} \end{array}\right\}$ and $\frac{13}{1000} \times \frac{5}{1000} = \frac{65}{1000000} = .000065.$

SCHOLIUM.

When a decimal or mixt Number is to be multiplied by 10, 100, 1000, 10000, &c. *'Tis only removing the separating Point in the Multiplicand so many Places towards the right Hand, as there are Cyphers in the Multiplier.*

$$.8546 \text{ mult. by} \left\{\begin{array}{l} 10 \\ 100 \\ 1000 \\ 10000 \end{array}\right\} \text{Product} \left\{\begin{array}{r} 8.546 \\ 85.46 \\ 854.6 \\ 8546 \end{array}\right.$$

EXAMPLE I.		EXAMPLE II.	
Mult.	38.546	Mult.	.54197
By	8.92	By	452
Product	343.83032	Product	244.97044

EXAMPLE III.

Mult.	.2349
By	.516
Product	.1212084

EXAMPLE IV.

Mult.	.154372
By	.2168
Product	.0333678496

EXAMPLE V.

Mult.	.005468
By	.0541
Product	.0002958188

EXAMPLE VI.

Mult.	.54198
By	.05416
Product	.0293536368

SCHOLIUM.

In *Multiplication*, if it were required to find only an affigned Part of the Product;

1. *Write the Unit's Place of the Multiplier under that Place of the Multiplicand, whofe Place you intend to keep in that Product, then invert the Order of all the other Figures*, that is, *write them in a contrary way.*

2. *Then in multiplying, always begin at that Figure in the Multiplicand which ftands over the Figures you are then multiplying withal, and fet down the firft Figure of each particular Product, directly one under another.*

3. *A due Regard muft be had to the Increafe, arifing from the Figure on the right Hand of that Figure in the Multiplicand, which you begin to multiply at.*

EXAMPLES.

Mult. 9.58 by 3.65.

Thus	9.58		9.58	Common way.
Inverted	56.3		3.65	
	2874	Prod.	34.9670	
	574			
	48			
	34.96			

Mult

Mult. 846.938 by 69.939.

Thus	846.938
Inverted	939.96
	5081628
	762245
	76224
	2540
	762
Product	59233.99

Multiply 3.141592 by 52.7438, for four Places of Decimals.

Thus	3.141592
Inverted	8347.25
	1570796
	62832
	21991
	1257
	94
	25
Product	165.6995

Multiply .234564 by .725234, for six, five, four Figures of the Product.

	.234564
Inverted	.432527
	.164194
	4691
	1172
	47
	7
Product	.170111

```
             .234564           .234564
    Inverted .432527           .432527
             -------           -------
               16421              1642
                 469                47
                 117                12
                   4           -------
             -------   Product   .1701
    Product   .17011
```

Multiply 104226.8672 by .261799388 for four Decimals.

```
    Thus  104226.8672
          .883997162
          -----------
            108453734
             62536120
              1042268
               729587
                93803
                 9379
                  312
                   83
                    8
          -----------
 Product   27286.5294
```

Multiply 419.3 by .6375 for the Integers only.

```
     419.3
    5736.0
    ------
    251
     13
      3
    ------
    267 required.
```

In the Product of 798.0625 by 78.54 for the Integers only.

```
798.0625
   45.87
   55864
    6384
     399
      31
   62679
```

The *Reason* of this Contraction is obvious.

For the *Index* of the *right-hand Figure* of any *Product* is the Sum of the *Indices* of the *Factors*.

And by inverting the Position of the Figures (as the Rule directs) the Sums of the Indices of each corresponding Place in the *Factors* will be equal among themselves, and therefore equal to the *Index* of the *right-hand Place* of the Product required.

But *Products*, whose *Indices* are equal, belong to the same *Place*; therefore must be set under each other, and *their Sum* must be the *Product* required.

These Contractions are of great Use and Facility in multiplying large Numbers by one another, also in resolving *adfected Equations*, or the calculating *Problems* in *Trigonometry* by the *natural Sines* and *Tangents*, &c. and in a vast variety of other Branches of Mathematics.

PROPOSITION IV.

DIVISION

Of Decimal Fractions.

RULE.

DIVIDE as if all were Integers (annexing Cyphers to the Dividend, if need be;) And

Let the first Figure in the Quotient be of the same Name (i. e. *have the same Index) with that Figure of the Dividend, which stands (or is imagin'd to stand) over the Unit Place of the Divisor.*

For the Index of each Figure in the Quotient must be equal to the Index of the divided Figure, less by the Index of the dividing Figure.

Or the Decimal Places in the Divisor and Quotient must be equal to those in the Dividend; if they are not, prefix Cyphers to the Quotient to supply the Defect.

For the Dividend is equal to the Product of the Divisor and Quotient; but both Factors contain as many Decimal Places, as the Product does:

Therefore, what Decimal Places are in the Dividend more than in the Divisor, must be supplied in the Quotient.

Thus .0416 divided by .26 gives .16 in the Quotient.

For $\left\{\begin{array}{l} .0416 = \frac{416}{10000} \\ .26 = \frac{26}{100} \end{array}\right\}$, and $\frac{26}{100})\frac{416}{10000}(= \frac{41600}{260000} = .16.$

SCHOLIUM I.

If the Divisor be greater, or have more Decimal Places, than the Dividend; then by annexing Cyphers to the Dividend, the Quotient may be had to any accuracy.

Thus .25).07864.00(.31456.

SCHOLIUM II.

Therefore, when there is a Remainder after Division, (tho'

(tho' neither Dividend or Divisor consist of any Decimals) it is but adding Cyphers to the Dividend, and proceed to any Exactness.

SCHOLIUM III.

When a decimal or mixed Number is to be divided by an Unit with Cyphers, it is but removing the Point, or Comma, in the Dividend, so many Places further towards the left Hand, as there are Cyphers annexed to the Unit, prefixing Cyphers to the Dividend to supply vacancy, if need be.

Thus,

		10		46.83
		100		4.683
468.3,	divided by	1000	Quotient is	.4683
		10000		.04683
		100000		.004683

SCHOLIUM IV.

In dividing by an infinite Number, the Division may oftentimes be very usually contracted, by

Taking as many of the left-hand Figures of the Divisor, as you think convenient, for the first Divisor, by which divide the given Number, and omit one Figure of the Divisor at each following Operation.

46876)9876403217(210692.

468.76)9876403217.00(21069210.7 &c.

Here, by adding as many Cyphers to the Dividend as are Decimals in the Divisor, the Quotient will be whole Numbers, and if any Remainder, add Cyphers, the rest are Decimals.

.46876)9876403217.00000(21069210710 &c.

Here I annex 5 Cyphers, as are the Decimals in the Divisor, and the Quotient is whole Numbers; if any Remainder, by adding Cyphers, the rest are Decimals.

468.76

468.76)98764.03217(210.692

.46876)98.76403217(210.692

46876).9876403217(.0000210692

468.76).9876403217(.00210692

.46876).9876403217(2.10692

I ſhall conclude Diviſion with inſerting only ſome few more Examples, wherein are contained all the Varieties that can happen in *Diviſion of Decimals*.

246)160.884(.654

246)16.0884(.0654

2.46)1608.84(654

2.46)16.0884(6.54

2.46)160884.00(65400

.0246)160.8840(6540

.0246).160884(6.54

PROPOSITION V.

TO reduce a Decimal into a Common Fraction.

RULE.

Multiply the Numerator of the Decimal by the Denominator of the Common Fraction.

Thus .875 of a Pound is 17*s.* 6*d.*

	.875	20
s.	17	500
		12
d.	6	000

: For 1000 : .875 : : 20*s.* : 17*s.* $\frac{5}{10}$.

; And 10 : .5 : : 12*d.* : 6*d.*

PRO-

PROPOSITION VI.

TO find a Multiplier that ſhall effect the ſame as a given Diviſor.

Case I.

If the Diviſor be an Integer, make it a Denominator, and 1 *the* Numerator of *that* Fraction *ſhall be the* Multiplier *required.*

For $\frac{N}{D}=N\times\frac{1}{D}$ or $\frac{3}{4}=3\times\frac{1}{4}$.

Case II.

If the Diviſor be a Fraction,
Its reverſe Fraction ſhall be the Multiplier required.

For $\frac{n}{d})\frac{N}{D}(=\frac{d}{n}\times\frac{N}{D}$, or $\frac{3}{4})\frac{2}{3}(=\frac{4}{3}\times\frac{2}{3}$.

Note, Hence, *Diviſor* given : 1 : : 1 : *Multiplier* required.

And by finding a Factor, when a Diviſor is given; or a *Diviſor*, when a *Factor* is given; one may advantageouſly contract ſeveral tedious Operations in *Arithmetic*, and find variety of excellent uſeful Rules for Eaſe and Expedition in *Common* and *Practical Accompts*.

Extraction of ROOTS.

ANY Number multiplied into itself is said to be squared; or the Square of that Number; *Example*, 4×4 =16, the Square of 4, or 4 squared. Also the Square of 12 is 144, for 12×12=144.

When a Number is given to be extracted for the Square Root, Sir *Isaac Newton* gives this

RULE.

Let it be first noted (says he) *with Points in every other Place, beginning from Unity; then write down such a Figure for the Quotient, or Root, whose Square shall be equal to, or nearest less than the Figure or Figures to the first Point. And then subtracting that Square, the other Figures of the Root will be found one by one, by dividing the Remainder by the Double of the Root, as far as extracted, and each Time taking from that Remainder the Square of the Figure that last came out, and the Decuple of the aforesaid Divisor augmented by that Divisor.*

Examples.

Extract the Square Root of 219024.

```
Thus      219024(468 the Root.
          16
       86)590
          516
      928)7424
          7424
             0
```

The Operation is thus performed: Find a Figure whose Square shall be equal to, or the next less Square to 21, which I find to be 4; then 4×4=16, which subtracted

tracted from 21 leaves 5; then bring down 90, the next Point, which being set on the right Hand of the 5, they make together 590; now to obtain the next Figure of the Root, you must divide this 590 by 8, the Double of the first Figure in the Quotient, saying, How oft is 8 contained in 59? You will find it to be 6 Times; wherefore place this 6 in the Quotient (being the second Figure of the Root) and place it on the right Hand of the Divisor, which makes it 86; then $86 \times 6 = 516$, which subtracted from 590 leaves 74, to which add the next Point 24, and it makes 7424, which you must divide by 92, the Double of the Root 46, saying, How many Times is 92 contained in 742? and you will find it to be 8 Times; then $928 \times 8 = 7424$, which subtracted will leave no Remainder; so the true Root is 468, as is easily proved by multiplying it by itself.

For $468 \times 468 = 219024$, the proposed Number.

What is the Square Root of 29506624?

```
Thus      29506624(5432 = the Root.
          25
     104)450
          416
    1083)3466
          3249
   10862)21724
          21724
          ....
```

Now, according to what I have said above, *viz.* when any Number is multiplied by itself, such *Product* thereby obtained is said to be the *Square* of that Number; and that Number, from whence such Square arises, is said to be the *Square Root* thereof; Therefore the Root multiplied into itself, will produce the Number proposed.

For $5432 \times 5432 = 29506624$, the proposed Number.

But it is impossible to extract the Root out of every proposed Number, for, there are Numbers *infinite*, which may be proposed, whose Square Roots cannot ex-

actly be expreſſed in finite Numbers; and ſuch are ſaid to be *ſurd*, or *irrational*.

To extract the Root of ſuch a Number, add two Cyphers to the Remainder, double the Root, and proceed as before, continuing to add to every Remainder two *Cyphers*, till you get 4 or 5 Places of Decimals in the Root, and ſo the Work may be carried on to any Degree of Nearneſs.

What is the Square Root of 26174983434?

```
Thus      26174983434(161786.84 the Root.
          1
          ---
     26)161
        156
        -----
     321)574
         321
         -----
    3227)25398
         22589
         -------
   32348)280934
         258784
         --------
  323566)2215034
         1941396
         ---------
 3235728)27363800
         25885824
         ----------
32357364)147797600
         129429456
         ----------
          18368144
         ----------
```

But ſince the Product 6×32356, or 1941396, ſubtracted from 2215034, leaves 273638, it is a ſign that the Number 161786 is not the Root of the propoſed Number 26174983434 preciſely, but that it is a little leſs. And in this Caſe, and in others like it, if you deſire the Root ſhould approach nearer, you muſt carry on the Operation in Decimals, by adding to the Remainder two Cyphers in each Operation. Thus the Remainder 273638, having two Cyphers added to it, becomes 27363800;

27363800; by the Division whereof by the Double of 161786, or 323572, you will have the first Decimal Figure 8. Then having writ 8 in the Quotient, subtract 8×3235728, or 25885824, from 27363800, and there will remain 1477976; and so by adding two Cyphers more, the Work may be carried on at pleasure.

When the proposed Number is a mixed one, point the Integers as above, and the Decimal Places, beginning in the second Place from the left-hand. An Example or two will make this familiar.

EXAMPLE.

What is the Square Root of 5479·32641?

First, add a Cypher to the Decimal Places, and it will make three compleat Points. Thus

```
      5479·326410(74.022 = the Root.
      49
  144)579
      576
14802)33264
      29604
148042)366010
       296084
        69926
```

SCHOLIUM.

As many Points as the Number given will admit of, so many Figures will the Square Root sought consist of.

What is the Square Root of 51?

First annex Cyphers at pleasure, and it will be 51·0000000000.

Then 51.000000000(7.14212 the Root.

```
        49
  141)200
      141
 1424)5900
      5696
14282)30400
       28564
142841)183600
       142841
1428422)4075900
        2856844
        1219056
```

To extract the Square Root of a Vulgar Fraction.

First, Extract the Square Root of the Numerator, and then of the Denominator; if there be no Remainder, set the Numerator over the Denominator so extracted, and the Thing is done.

The Square Root of $\frac{25}{36}$ is $\frac{5}{6}$;

```
For 25(5 Numerator      36(6 Denominator.
    25                  36
    ..                  ..
```

Therefore $\frac{5}{6}$ is the Root required.

But when you cannot extract the Root of a Vulgar Fraction without a Remainder, put the Fraction into a Decimal, and extract the Root thereof, as above.

What is the Square Root of $\frac{29}{37}$?

Thus $\frac{29.000000}{37} = .783778.$

And .783778(.826 the Root.

```
      64
  162)437
      324
 1646)11378
       9876
       1502
```

But

But when the Root is carried on half way, or above, the rest of the Figures may be obtained by Division alone; as in this Example, if you had a mind to extract the Root to 5 Figures, after the three former .826 are extracted, the two latter may be had by dividing the Remainder by the Double of .826.

And after this manner, if the Root of 46945 was to be extracted to five Places in Numbers; after the Figures are pointed, write 2 in the Quotient, as being the Figure whose Square, =4, is the greatest contained in 4 (or 4 exactly) the Figure to the first Point; and having taken the Square of 2 from 4, there will remain 0; then having set the two next Figures, *viz.* 69, see how many times the Double of 2, or 4, is contained in 6; and you will see at first sight no more than 1; having wrote 1 in the Quotient, and subtracted 41×1, or 41, from 69, there will remain 28; and having set down to this the Figures 45, see how many times the Double of 21, or 42, is contained in 284, and you will find 6; then write 6 in the Quotient, and having subtracted 6×426, or 2556, there will remain 289. Lastly, to obtain the remaining Figures, divide the Remainder 289 by the Double of 216, or 432, and you will have the Figures .66, which being writ in the Quotient, you will have the Root 216.66.

After the same manner Roots are also extracted out of Decimal Numbers.

Thus	2691.90(51.8835	0.432190(0.65745
	25	0
	101)191	43
	101	36
	1028)9090	125)721
	8224	625
	1036)8660(835	130)969(745

```
    4.3219(2.0789
    4
407)3219
    2849
 414)3700(89
```

CUBE ROOT.

As any Number, or Root, multiplied into itſelf, produces a Square, or ſecond *Power*, ſo again that *Product*, or *Square*, multiplied into the ſame Root, produces a *Cube*, or third Power; and that Number from whence ſuch a Cube ariſes, is the *Cube Root* thereof, *&c.* as is manifeſt from the following Table of Powers.

Roots	1	2	3	4	5	6	7	8
Squares	1	4	9	16	25	36	49	64
Cubes	1	8	27	64	125	216	343	512
Biquadrats	1	16	81	256	625	1296	2401	4096
Surſolids	1	32	243	1024	3125	7776	16807	32768
Square cub'd	1	64	729	4096	15625	46656	117649	262144
Second Surſolids	1	128	2187	16384	78125	279936	823543	2097152
Biquadrats ſquar'd	1	256	6561	65536	390625	1679616	5764801	16777216
Cubes cub'd	1	512	19683	262144	1953125	10077696	40353607	134217728

Note,

Note, For eafe in the following Operations, I make ufe of this Notation, *viz.*

$\overline{9|}^{2} = 81$, or the Square of 9.

$\overline{9|}^{3} = 729$, or the Cube of 9.

$\overline{9|}^{4} = 6561$, or 9 raifed to the fourth Power; and fo of any other Number or Numbers whatever.

The Extraction of the Cubic Root, and of all others, Sir *Iſaac Newton* comprehends under this one general

RULE.

Every third Figure beginning from Unity is firſt of all to be pointed, if the Root to be extracted be a Cubic one; *or every fifth, if it be a* Quadrato-Cubic, *or of the fifth Power*, &c. *and then ſuch a Figure is to be writ in the Quotient*, whofe *greateſt Power* (i. e. *whoſe* Cube, *if it be a* Cubic *Power*; *or whoſe* Quadrato-Cube, *if it be the* fifth *Power*, &c.) *ſhall either be equal to the Figure or Figures before the firſt Point, or the next leſs; and then having ſubtracted that Power, the next Figure will be found by dividing the Remainder, augmented by the next Figure of the Reſolvend, by the next greateſt Power of the Quotient, multiplied by the* Index *of the Power to be extracted*; *that is, by the* triple Square *of the Quotient, if the Root be a Cubic one*; *or by the* quintuple Biquadrate, *i. e.* five times the Biquadrate, *if the Root be of the* fifth Power, &c. *And having again ſubtracted the greateſt Power of the whole Quotient from the firſt Reſolvend, the third Figure will be found by dividing that Remainder, augmented by the next Figure of the Reſolvend, by the next greateſt Power of the whole Quotient, multiplied by the* Index *of the Power to be extracted*; and fo on *in infinitum.*

Thus to extract the Cube Root of 13312053, the Number is firft to be pointed after this Manner, *viz.* $1\dot{3}31\dot{2}05\dot{3}$. Then you are to write in the Quotient the Figure 2, whofe Cube 8 is the next lefs Cube to the Figures

gures 13 (which is not a perfect Cube Number) or to the first Point; and having subtracted that Cube, there will remain 5; which being augmented by the next Figure of the Resolvend 3, and divided by the triple Square of the Quotient 2, by seeking how many times 3×4, or 12, is contained in 53, it gives 4 for the second Figure of the Quotient; but since the Cube of the Quotient 24, *viz.* 13824, would come out too great to be subtracted from 13312 that precedes the second Point, there must only 3 be written in the Quotient. Then 23 cub'd, that is, 23×23=529×23=12167 the Cube, taken from 13312 will leave 1145; which augmented by the next Figure of the Resolvend 0, and divided by the triple Square of the Quotient 23, *viz.* by seeking how many times 3×529, or 1587, is contained in 11450, it gives 7 for the third Figure of the Quotient. Then the Quotient 237 multiplied by 237 gives the Square 56169, which again multiplied by 237 gives the Cube 13312053, and this taken from the Resolvend leaves 0. Whence it is evident that the Root sought is 237. See *Newton's Arithmetica Universalis.*

	$\dot{1}33\dot{1}205\dot{3}$(237 Root
Subtract the Cube	8
2×2=4×3=12)	53(4 or 3
Subtract Cube	12167
1587) Rem.	11450(7
Subt. Cube	13312053
Remains	0

EXAMPLES.

To extract the Cube Root of 314432?

Thus	$31\dot{4}43\dot{2}$(68 Root.
	216
For, 6×6=36×3=108)	984(8
And, 68×68=4624×68=	314432
	0

To

To extract the Cube Root of 5735339.

Thus 5735339(179 Root
Subtract Cube 1

$$
\begin{array}{lr}
 & 3)\underline{47(7} \\
\overline{17}|^3 = & \underline{4913} \\
17\times17=289\times3=867) & \underline{8223(9} \\
\overline{179}|^3 = & \underline{5735339} \\
 & 0
\end{array}
$$

To extract the Cube Root of 32461759.

Thus 32461759(319 Root
Subtract Cube 27

$$
\begin{array}{lr}
3\times3=9\times3=27) & \underline{54(1} \\
\overline{31}|^3 = & \underline{29791} \\
31\times31=961\times3=2883) & \underline{26707(9} \\
\overline{319}|^3 = & \underline{32461759} \\
 & 0
\end{array}
$$

To extract the Cube Root of 22069810125.

Thus 22069810125(2805 Root
8

$$
\begin{array}{lr}
2\times2=4\times3=12) & 140(8 \\
\overline{28}|^3 = & \underline{21952} \\
28\times28=784\times3=2352) & 11781(05 \\
\overline{2805}|^3 = & \underline{22069810125} \\
 & 0
\end{array}
$$

To

To extract the Cube Root of 25917056.

Thus 25917056(295.9 Root

8

$2 \times 2 = 4 \times 3 = 12)$ 179(9

$\overline{29}|^3 =$ 24389

$\overline{29}|^2 = 841 \times 3 = 2523)$ 15280(5

$\overline{295}|^3 =$ 25672375

$\overline{295}|^2 = 87025 \times 3 = 261075)$ 244681,0... (.9

$\overline{295.9}|^3 =$ 25908060079

8995921 Remainder.

And so by adding to the Remainder 3 Cyphers, the Operation may be carried on to any required Places of Decimals.

To extract the Cube Root of 1881365963625.

Thus 1881365963625(12345 Root.

1

3) 8 (2

1728

432) 1533 (3

1860867

45387) 204989(4

1879080904

4568268) 22850596(5

1881365963625

0 0 0

S C H O L I U M.

1. In Square and Cube Numbers, there are certain Properties worth the Reader's Attention; as, if from an Unit you successively add the odd Numbers, all those Numbers shall be square Numbers; as if to 1, you add the next odd Number, which is 3, the Sum is 4, a Square Number; to which add 5, the next odd Number, and the Sum is 9, a Square Number also; to which

 add

add 7 the next odd Number, and the Sum is 16, a Square Number alſo, and ſo on.

2. Likewiſe, if Cubic Numbers be ſucceſſively added from Unity, theſe Numbers will be alſo Square Numbers.

To extract the Cube Root of a Vulgar Fraction.

Reduce the Fraction to its loweſt Terms, and extract the Cube Roots of the *Numerator* and *Denominator*, for a new *Numerator* and *Denominator*. But if the *Fraction* be a *Surd*, reduce it to a *Decimal*, always remembering to let your *Decimal Fraction* conſiſt of *Ternaries* of Places, as *three*, *ſix*, *nine*, *twelve*, &c.

EXAMPLES.

The Cube Root of $\frac{1944}{4608}$ is $=\frac{3}{4}$.

For $\frac{1944}{4608}$ is $=\frac{27}{64}$, and the Cube Root of $\frac{27}{64}$ is $\frac{3}{4}$.

The Cube Root of $\frac{512}{1000}$ is $=\frac{4}{5}$.

For $\frac{512}{1000}$ is $=\frac{64}{125}$, and the Cube Root is $\frac{4}{5}$; and ſo of any other.

To extract the Cube Root of $\frac{115}{321}$.

Here the propoſed Fraction being a Surd, reduce it into a Decimal Fraction, which is $=\cdot358255$.

Then ·358̇255̇(·71, the Root of $\frac{115}{321}$.

343

147)152(1

357911

344

Which may be carried on to any Degree of exactneſs by adding ternary Cyphers.

BIQUA-

BIQUADRATE ROOT.

Any Number involved *four* times produces a *Biquadrate*. Therefore to extract the *Biquadrate* Root of any proposed Number is no more than extracting twice the Square Root, *viz.* the Square Root of the first Root.

EXAMPLES.

Extract the Biquadrate Root of 4857532416.

```
Thus     4857532416(69696
         36
    129)1257
        1161
     1386)9653
          8316
    13929)133724
          125361
     139386)836316
            836316
              0
```

```
And   69696(264, the Biquadrate Root.
      4
   46)296
      276
  524)2096
      2096
        0
```

For $264 \times 264 \times 264 \times 264 = 4857532416$.

To extract the Biquadrate Root of 296637086736.

Thus $\sqrt{296637086736} = 544644$.

And $\sqrt{544644} = 738$ the Biquadrate Root.

Note, The above may be expressed thus:

$\sqrt{:\sqrt{296637086736}}$; or thus, $\overline{\sqrt{296637086736}}\big|^{\frac{1}{2}}$; or thus, $\overline{296637086736}\big|^{\frac{1}{4}} = 738$.

Alſo the Biquadrate Root of 99887766554411 is = 3161.38.

For $\sqrt{99887766554411} = 9994381.7494$.

And $\sqrt{9994381.7494} = 3161.38$.

And ſo of any other.

QUADRATO-CUBCAL ROOT, or SURSOLID.

ANY Number involved *five* times produces a Quadrato-Cube, and the Number thereof is the Root.

To extract the Quadrato-Cubical Roo of 36430820, it must be pointed over every *fith* Figure, thus, 36430820 and the Figure 3, whose Quadrato-Cube (or fifth Power) 243 is the next less to 364, *viz.* to the first Point, must be writ in the Quotient; as appears by inspection of the Table. Then the Quadrato-Cube 243 being subtracted from 364, there remains 121, which augmented by the next Figure of the Resolvend, *viz.* 3, and divided by five times the Biquadrate of the Quotient, *viz.* by seeking how many times 5x81, or 405, is contained in 1213, it gives 2 for the second Figure. That Quotient 32, being thrice multiplied by itself, makes the Biquadrate 1048576; and this again multiplied by 32 makes the Quadrato-Cube 33554432, which being subtracted from the Resolvend leaves 2876388. Therefore 32 is the Integer Part of the Root, but not the exact Root; wherefore, if you have a Mind to prosecute the Work in Decimals, the Remainder, augmented by a Cypher, must be divided by five times the aforesaid Biquadrate of the Quotient, by seeking how many times 5x1048576, or 5242880, is contained in 2876388,0, and there will come out the third Figure, or the first Decimal 5; and so by subtracting the Quadrato-Cube of the Quotient 32.5 from the Resolvend, and dividing the Remainder by five times its Biquadrate, the fourth Figure may be obtained.

And so on *in infinitum*. *See* Newton's *Arithmetica Universalis*.

OPERATION.

36430820(32.5

243

405)1213(2

33554432

5242880) 2876388,2(5

To extract the Quadrato or Cubical Root of 102434508843424.

Thus 102434508843424(634

7776

6480)24674(3

992424636

78763860)319204524(4

102434508843424

Of ALGEBRA.

ALGEBRA is a Science of universal Quantity, whereby difficult Questions in Arithmetic and Geometry may be solved; and is called the *Analytic Art*, and may be defined to be a *general Method of Reasoning, in which by assuming the Quantity sought as if it were known*, by the help of one or more Quantities really known, we proceed by *Addition*, *Subtraction*, *Multiplication*, or *Division*, till at last the Quantity sought is found equal to some real known Quantity, and so itself comes to be discovered.

In this Art Quantities are represented by Letters, and sometimes by Figures, if there be occasion; and these are either Affirmative or Negative. An affirmative Quantity is denoted by the Sign $+$, and is defined to be whatsoever is greater than nothing. A negative Quantity is denoted by the sign $-$, and signifies a Quantity less than nothing. The Meaning whereof is, that, whereas all contrary Quantities meet in one common Limit, which equally partakes of both Extreams, and is always represented by a Cypher or 0, if all Quantities on one side the Limit be considered as Affirmative, then all those on the contrary side ought to be look'd upon as Negative. These Signs always belong to the Quantities immediately following them; and all Quantities with these two Signs are always to be interpreted in a contrary Signification: If $+$ signifies upward, forward, above, before, gain, increase, addition, &c. then $-$ is to be interpreted, downwards, backwards, below, behind, loss, decrease, subduction, &c. And if $+$ be understood of these, then $-$ is to be interpreted of the contrary. And as these affirmative and negative Quantities are contrary to one another in their own Natures, so likewise are they in their Effects; which Considera-

tion

tion alone, if duly attended to, would anfwer all the Queftions concerning the Signs of Quantities, arifing from their Addition, Subtraction, Multiplication, &c.

This Art is very defervedly reputed the very Apex of human Learning; for, by means thereof, furprifing Truths have been found, as well in natural Philofophy, as pure Mathematics. By this many geometrical Demonftrations are wonderfully abridg'd, and Problems folved, which would be otherwife impoffible to be effected; nay even fuch a Number of Truths is often expreffed in one Line by this Art, as would require a whole Volume to expound and demonftrate, otherwife; and by contemplating one Line for a few Hours, you may learn what would take up a whole Year to be learned according to the common Methods.

NOTATION *of* ALGEBRA.

1. Letters of the Alphabet may ftand for any Number or Quantity whatever, whether known or unknown. Though it is ufual to put the firft Letters of the Alphabet, *a*, *b*, *c*, &c. for known Quantities; and the laft *x*, *y*, *z*, &c. for unknown ones. Though fome put Vowels *a*, *e*, *y*, &c. for unknown Quantities.

2. In Operations perform'd by *Literal Arithmetic*, all Quantities, *known*; or *unknown*, are reprefented by Letters, with prefix'd *Marks* or *Signs*, whereby (according to the Nature of the *Propofition)* they may be fo ordered by Addition, Subtraction, &c. as if each particular Part was actually known; fo that the Pofition and Relation of one Quantity to another are vifible thro' the whole Courfe of the Procefs, and confequently that of the *known* to the *unknown*.

3. The Number of times any *Quantity* is taken muft be prefix'd to it, and is called a *Co-efficient*, or *Co-factor*; as $2a$ denotes $2a$'s, $5b$ five b's, $25x$ twenty five x's.

4. A *Quantity* having no Sign before it, muft always be taken to be Affirmative; and if it hath no numeral Coefficient before it, *Unity* muft always be underftood; as a is $1a$, x^2 is $1x^2$, &c.

5. *Simple*

5. *Simple* Quantities are those that have but one Member; as $3b$, $2x$, *&c.* *Compound* Quantities are those which are connected by $+$ and $-$; as $5a+ba-m-y+g$.

6. Two or more Species, immediately connected together, denote a *Product*, or Quantity made by the *Multiplication* of all the Species together. Thus ab denotes a Quantity made by multiplying a by b, and $bmxy$ denotes $b \times m \times x \times y = bmxy$; the Product arising from b multiplied by m, and that Product by x, and the Product again by y; but the chief Use of these Notes is, when compound Quantities, as $m-x$, were to be multiplied by $m+3x$. The Way is to draw a Line over each Quantity, and write them thus: $\overline{m-x} \times \overline{m+3x}$. Also $3a+4b-zx$ to be multiplied by $mz-zy+mx+10y^2$, is to be wrote thus: $\overline{3a+4b-zx} \times \overline{mz-zy+mx+10y^2}$; and so of any other.

7. One *Quantity* below another, with a Line interposed, denotes a *Quotient*, or a Quantity, arising by the Division of the upper Quantity by the lower. As $\frac{m}{n}$ denotes a Quantity which arises by dividing m by n. Likewise thus: $\frac{xx-yy}{m+r}$ denotes a Quantity arising by dividing $xx-yy$ by $m+r$; and so in others. Sometimes the Divisor is set before the divided Quantity; as thus: $\overline{m+r})xx-yy$.

8. If a Quantity be multiplied by itself, the Number of *Facts*, or Products, is, for shortness sake, set at the top of the Letter. Thus for xx we write x^2; for $xxxx$, x^4; for $mmmaaabbxx$ we write $m^3a^3b^2x^2$. And these Numbers standing above the Letters are called Indices; thus 4 is the Index of x in the Quantity x^4.

9. $\frac{2}{3}a^2b^3x$ denotes two thirds of a^2b^3x, and $5\frac{a}{b}$ signifies five times $\frac{a}{b}$, and $11\sqrt{mx}$, eleven times $\sqrt{mx}$. Also

m-

$\frac{m}{n}z$ denotes the Product of z by $\frac{m}{n}$; and $\frac{24xx}{14a+by}y^5$ denotes the Product made by multiplying y^5 by $\frac{24xx}{14a+by}$, that is, y^5 multiplied into the Quotient arising by the Division of $24xx$ by $14a+by$; and $\sqrt{\frac{24xx}{14a+by}y^5}$ denotes the Sq. Root thereof; and $\overline{5a+3x}\sqrt{mx+zy+r^2}$ denotes the Product of $\overline{5a+3x}$ into the Root $\sqrt{mz+zy+r^2}$; and $\frac{myz}{\sqrt{\frac{1}{4}aa+bb}}$ denotes the Quantity myz divided by the Root $\sqrt{\frac{1}{4}aa+bb}$; and $\sqrt{bb-c}.\sqrt{xy+z}$ denotes the Root $\sqrt{bb-c}$ multiplied into the Root $\sqrt{xy+z}$; and $\sqrt{xy\sqrt{mz+x}}$ denotes the Root of the Product of the Quantities xy into $\sqrt{mz+x}$; and so in other Cases.

And since *Addition*, *Subtraction*, *Multiplication*, and *Division*, are the common Affections of all *Quantities*, therefore we shall, in the next Place, endeavour, with all the Brevity and Plainness possible, to apply these Rules to *Letters*.

AXIOMS.

1. If *to* or *from* equal Quantities, equal ones be *added* or *subtracted*, their *Sum* or *Remainder* will be equal.

2. If equal Quantities be *multiplied* or *divided* by equal ones, their *Products* or *Quotients* will be equal.

3. If *from* the Sum of any two Quantities be taken either of them, the *Remainder* will give the other Quantity.

4. The Difference of any two Quantities *added* to the *less* gives the greater Quantity, but *subtracted* from the *greater* Quantity gives the *lesser*.

5. The *Product* of any two Quantities divided by either of them, the *Quotient* arising from thence will give the other Quantity.

6. The *Quotient* of any two Quantities being *multiplied* by the *lesser*, the *Product* is the *greater* Quantity.

7. Quan-

7. Quantities equal to a Third are equal to one another.

Note, *It would be very proper for the Learner to get these Axioms by heart, as he will find them to be of great use in solving Problems.*

ADDI-

ADDITION of ALGEBRAIC INTEGERS.

CASE I.

WHEN Quantities have the same Name, *and the same* Signs;

RULE.

Put down the Sum of their numeral Coefficients, with their common Sign before it, and the common Denomination after it.

EXAMPLES.

To	$+a$	$+13b$	$+m$	$-5r$	$xx+yy-2x+5$
Add	$+a$	$+5b$	$+15m$	$-r$	$xx+2yy-3x+12$
Sum	$2a$	$18b$	$16m$	$-6r$	$2xx+3yy-5x+17$

For it is evident from the common Way of numbering, that $13+5=18$ of any thing of like Name; as 13 Yards and 5 Yards are 18 Yards.

	1	$+a$	$11m$	n	$r+s$
	2	$+3a$	$19m$	n	$r+s$
	3	$+5a$	$3m$	n	$r+s$
	4	$+9a$	m	n	$r+s$
Sum	5	$18a$	$33m$	$4n$	$4r+4s$

$$\begin{array}{r} xz-yu-m+b-11 \\ 3xz-yu-m+3b-19 \\ 2xz-3yu-7m+b-18 \\ xz-2yu-m+b-54 \\ \hline 7xz-7yu-10m+6b-102 \end{array}$$

CASE II.

When the Quantities have the same Name, *and Different* Signs,

RULE.

Put only the Difference of their Coefficients with the common Denomination after it, and the Sign of the greater Quantity before it.

Ex-

EXAMPLES.

To	$+5a$	$-7x$	$-5xy$
Add	$-3a$	$+3x$	$+5xy$
Sum	$+5a-3a$	$-7x+3x$	$+5xy-5xy$
Or	$+2a$	$-4x$	0

To	$11a^2-y+50-x$
Add	$y-40-11a^2+x$
Sum	$11a^2-11a^2-y+y-x+x+50-40$
Or	10

For to add a *Negative* is to take away a *Positive*.

Therefore to connect a *Negative* and a *Positive*, is to make the one, and destroy the other.

Thus, if *A.* has 1000*l.* and owes 800*l.* it is evident that the Sum, or his Worth, is but 200*l.*

And if *A.* has 800*l.* and owes 1000*l.* then his Worth is —200*l.* or 200*l.* in debt, or worse than nothing, because he owes 200*l.* more than he actually can pay.

To	$+y$	$-11xy$	$-x$	$za-11+2x+3y-a$
Add	$-3y$	$-21xy$	$+x$	$-3x-5a-za+23-3y$
Add	$-2y$	$+50xy$	$-2x$	$-3y+2x-11a+42-3za$
Add	$+4y$	$-15xy$	$+2x$	$+3a+4za-2y-21-4x$
	0	$+3xy$	0	$+za+33-3x-5y-14a$

In ordering such Examples as the above, set down the Quantities having the same Sign, whether affirmative or negative, on a separate Bit of Paper, or a Slate; then gather those of a different Sign, and compare them together, which must be set down according to the Prescript of the Rule.

$$\text{Thus}\quad \begin{array}{r} +za \\ +4za \\ \hline +5za \end{array} \qquad \begin{array}{r} -za \\ -3za \\ \hline -4za \end{array}$$

It is evident that $+5za$ *less* $4za$, the Sum is $+za$, and so any other.

CASE III.

When Quantities are of different Names.

RULE.

When the Quantities to be added are of different Denominations, and consequently such as will not incorporate, they can only be put down in one common Series, with their proper Signs before them. So compound Quantities, whose Numbers are all of different Denominations, are incapable of being added any other way, than by being placed one after another without altering the Signs; but if the Members are not all of different Denominations, it may then be convenient to place one compound Quantity under another, with like Parts under like, as far as possible, and then add them together.

EXAMPLES.

To	a	$3a$	$a+x$	$2a-3x-4y+30$
Add	b	$2m$	$b+m$	$24+bb+4a+3x$
Sum	$a+b$	$3a+2m$	$a+x+b+m$	$6a+bb-4y+54$

This sort of Addition is very plain, that *a* added to *b* cannot be *ab*, but $a+b$; for suppose *a* stood for 12 Yards, and *b* for 6 Shillings, it will neither make 18 Yards, nor 18 Shillings, but 12 Yards *plus* 6 Shillings.

Again, suppose *a* denotes 5 Pounds, and *b* four Gallons, yet it will not make 9 Pounds, nor 9 Gallons; for 5 Pounds will stand as 5 Pounds, and so will 4 Gallons stand as 4 Gallons; therefore it is manifest they cannot be added together but by connecting the Signs.

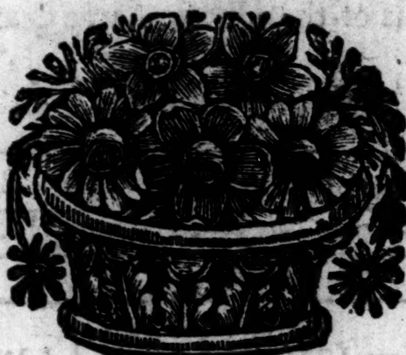
Here follow a few Examples promiscuously set to exercise the Learner in the above three Cases of *Addition*, which, when well understood, he may proceed to *Subtraction*.

$$\begin{array}{rl} & 3y+ab-m+2z \\ & 2y-ab+m+5zz \\ & 15-21y+9m-3x \\ \hline \text{Sum} & 5zz+2z-16y+15+9m-3x \end{array}$$

$$\begin{array}{rl} & 24+3x+aa-bx+yy \\ & 3y+4d+xx+bx-yy \\ & -xx-24-3x+5b-7yy \\ \hline \text{Sum} & aa+3y+4d+5b-7yy \end{array}$$

$$\begin{array}{rl|l} & aa+2ax+xx & 8by^2+bcx-37y \\ & -4ax & 7by^2-bcx+54y-11 \\ \hline \text{Sum} & aa-2ax+xx & 15by^2+17y-11 \end{array}$$

$$\begin{array}{rl} & 5mx+11yz-54xy-54xx-11 \\ & -3mx-11zy+19nx+9zz-11 \\ & -15mx+54xx+17zz+54xy+21 \\ \hline \text{Sum} & +6mx+26zz-1 \end{array}$$



Subtraction of Algebraic Integers.

WHENEVER a simple *Algebraic* Quantity is to be subtracted from another, whether simple or compound, first change the Sign of the Quantity to be subtracted, that is, if it be affirmative, make it, or at least call it, negative, and *vice versa*, and then add it so changed to the other. For since, as was before hinted, the subtracting of any one Quantity from another is the same in effect as adding the contrary, and since changing the Sign of the Quantity to be subtracted renders it contrary to what it was before, it is evident that after such a Change it may be added to the other, and that the Result of this Addition will be the same with that of the intended Subtraction. Thus may the Rule for Subtraction, by changing the Sign of the Quantity to be subtracted, be any time changed into that of Addition, just as the Rule for Division of Fractions, by inverting the Terms of the Divisor, was changed into that of Multiplication.

EXAMPLES.

From	$+5a$	$+bc$	$-2x$	$11m+12x+13$
Take	$+3a$	$+bc$	$-2x$	$50\ +11x+29$
Remains	$+5a-3a$	$bc-bc$	$-2x+2x$	$11m+x-66$
Or	$2a$	0	0	

From	$+5a$	$-12m$	$+9x$	$13xx-12x+20$
Take	$-3a$	$+11m$	$-6x-z$	$xx+x-12$
Remains	$+8a$	$-23m$	$+15x+z$	$12xx-13x+32$

From	$12x+6a-4b-12c\ *\ -7e-5f$
Take	$10x+9a+4b-5c+6d-7e\ *$
Remains	$+2x-3a-8b-7c-6d\ *\ -5f$

SCHO-

SCHOLIUM.

As to Order, how the ſeveral Members or Terms do ſtand, it matters not in *Addition* and *Subtraction*, ſo that each has its own peculiar Sign.

For $a+m-y=a-y+m=-y+m+a=m-y+a=s$;
Or $6+8-4=6-4+8=-4+8+6=8-4+6=10$.

Multiplication of Algebraic Integers.

BEFORE we can proceed to the Multiplication of *Algebraic* Quantities, we are to take notice, that if the Signs of the *Multiplicator* and *Multiplicand* be both alike, that is, both affirmative, or both negative, the Product will be affirmative, otherwise negative.

Thus $+4$ multiplied into $+3$, or -4 into -3, produces in either Case $+12$. But -4 multiplied into $+3$, or $+4$ into -3, produces in either Case -12; all which four Cases are thus demonstrated.

First, that $+4$ multiplied into $+3$, produces $+12$, and 2dly, that -4 into $+3$ gives -12, is evident from the Nature of Multiplication by an Affirmative, which is but a more compendious Addition. Thirdly, that $+4$ into -3 gives -12, is evident from hence, that $+4$ multiplied into the Progression 3, 2, 1, 0, produces the Progression 12, 8, 4, 0; therefore $+4$ multiplied into the rest of the Progressions of Multiplicators, *viz.* -1, -2, -3, ought to produce the rest of the Progression of Products, *viz.* -4, -8, -12. Fourthly, that -4 multiplied into -3 produces $+12$ will be evident from the like way of reasoning; for -4 multipled into the Progression 3, 2, 1, 0, produces the Progression -12, -8, -4, 0, by the second Case; therefore -4 multiplied into a Continuation of the former Progression, viz. -1, -2, -3, ought to produce a Continuation of the latter Progression, $+4$, $+8$, $+12$; or, more compendiously, thus: $+4$ into $+3$ produces $+12$, therefore $+4$ into -3 (which is contrary to $+3$) ought to produce something contrary to $+12$, that is, -12. But if $+4$ into -3 produces -12, then -4 into -3 ought to produce something contrary to -12, namely $+12$. And thus the formidable Paradox -4 into -3 produces $+12$ is found at last to amount to no more than a common Principle in *Grammar*, viz. *That two Negatives make an Affirmative.*

1. $+4$

1. $+4$ into $+3$ gives $+12$
2. $+4$ into -3 gives -12
3. -4 into $+3$ gives -12
4. -4 into -3 gives $+12$.

Multiply	4,	4,	4,	4,	4,	4,	4
Into	3,	2,	1,	*,	-1,	-2,	-3
	12,	8,	4,	*,	-4.	-8,	-12

Multiply	-4,	-4,	-4,	-4,	-4,	-4,	-4
Into	$+3$,	$+2$,	$+1$,	*,	-1,	-2,	-3
	-12,	-8,	-4,	*,	$+4$,	$+8$,	$+12$

These Things premised, the Multiplication of simple Algebraic Quantities is performed, first by multiplying their numeral Coefficients together, and then putting down after the Product all the Letters in both Factors, the Sign, when occasion requires, being prefixed as above directed. Thus $5x \times 4a = 20ax$.

Tho' this Kind of Language (for it is no more) like all others, be purely arbitrary, yet that a more rational one could not have been invented for this Purpose, will appear by the following Consideration. If any Quantity, as x, is to be multiplied by any Numbers, as 2, 3, 4, &c. the Product can't be better represented than by $2x$, $3x$, $4x$, &c. Therefore if x is to be multiplied by a, the Product ought to be called ax; but if x multiplied into a produce ax, then $4x$ multiplied into a, ought to produce 4 times as much, that is, $4ax$: Lastly if $4x$ multiplied into a produce $4ax$, then $4x \times 5a = 20ax =$ five times as much.

Distinctions to be observed betwixt Addition *and* Multiplication.

1.	a added to $a =$	$2a$,	but	$a \times a = aa$
2.	a added to $* =$	a,	but	$a \times * = *$
3.	a added to $-a =$	*,	but	$a \times -a = -aa$
4.	$-a$ added to $-a =$	$-2a$,	but	$-a \times -a = +aa$
5.	a added to $1 =$	$a+1$,	but	$a \times 1 = a$
6.	$2a$ added to $-3b =$	$2a-3b$,	but	$2a \times -3b = -6ab$

PROB-

PROBLEM.

TO multiply simple Quantities.

RULE.

Join the Factors together, and prefix to them the Product of the Coefficients, if there be any.

EXAMPLES.

Multiply	$4bm$	$-gx$	$-2cd$	$-x$	$-\frac{1}{2}x$
By	$3m$	$4rt$	$15xy$	$-x$	$-\frac{1}{2}x$
Product	$12bm^2$	$-4grtx$	$-30cdxy$	$+xx$	$+\frac{1}{4}xx$

PROBLEM II.

TO multiply compound Quantities.

RULE.

Multiply each Part of the Multiplier into each Part of the Multiplicand.

EXAMPLES *of Compounds by Simples.*

Multiply	$a+b$	$x-y$	$mn-41$
By	m	y	9
Product	$ma+mb$	$xy-yy$	$9mn-369$

Multiply	$x-y+mx$
By	$-my$
Product	$-mxy+myy-m^2yx$

EXAMPLES *of Compounds by Compounds.*

Multiply	$a+x$	$3y-m+8$
By	$a-x$	$y-9$
	$aa+ax$	$3yy-my+8y$
	$-ax-xx$	$-27y+9m-72$
Product	$aa-xx$	$3yy-my-19y+9m-72$

Multi-

$$
\begin{array}{lllllll}
\text{Multiply} & 8x^3+5x^2-3x+1 & & & & & \\
\text{By} & x^2-5x+9 & & & & & \\
\hline
 & 8x^5 & +5x^4 & -3x^3 & +x^2 & & \\
 & & -40x^4 & -25x^3 & +15x^2 & -5x & \\
 & & & +72x^3 & +45x^2 & -27x & +9 \\
\hline
\text{Product} & 8x^5 & -35x^4 & +44x^3 & +61x^2 & -32x & +9 \\
\hline
\end{array}
$$

$$
\begin{array}{ll}
\text{Multiply} & 3z^2x+myy-zz+24-19x \\
 & a^7+x^3z-11+21y^5 \\
\hline
 & 3a^7z^2x+a^7myy-a^7zz+24a^7-19a^7x \\
 & \quad +3x^4z^3+my^2zx^3-x^3z^3+24x^3z-19x^4z \\
 & \quad\quad -33z^2x-11my^2+11zz-264+209x \\
 & \quad\quad\quad +63z^2xy^5+21my^7-21z^2y^5+504y^5-399xy^5 \\
\hline
\text{Product} & 3a^7z^2x+a^7my^2-a^7z^2+24a^7-19a^7x+3x^4z^3+my^2zx^3-x^3z^3+24x^3z \\
 & -19x^4z-33z^2x-11my^2+11zz-264+209x+63z^2xy^5+21my^7 \\
 & -21z^2y^5+504y^5-399xy^5. \\
\hline
\end{array}
$$

SCHO-

SCHOLIUM.

Sometimes Products are expressed only by the Quantities to be *multiplied* with the Sign $\times$ between them.

Thus the Product $m+x$ by $n+y$ is $\overline{m+x}\times\overline{n+y}$, and the Product $a+x$ by $m-n+y$, and that Product by $b+rx$ is $\overline{a+x}\times\overline{m-n+y}\times\overline{b+rx}$.

Division of Algebraic Integers.

General RULE.

PUT the Divisor under the Dividend, with a Line between them.

EXAMPLES.

Divide	a	c	a^2+m	$a+dg-m^2$	$m-x+ab$
By	b	x	$x-g$	$y+z^2$	$r-n+c$
Quotient	$\frac{a}{b}$	$\frac{c}{x}$	$\frac{a^2+m}{x-g}$	$\frac{a+dg-m^2}{y+z^2}$	$\frac{m-x+ab}{r-n+c}$

For, by *common Division*, in dividing 12 by 4, it is the very ſame Thing whether 3 or $\frac{12}{4}$ be the Quotient.

The Diviſion of ſimple Algebraic Quantities (where it is poſſible) is performed, firſt, by dividing the numeral Coefficient of the Dividend by the numeral Coefficient of the Diviſor, and then putting down after the Quotient all the Letters in the Dividend which were not in the Diviſor, the Sign of the Quotient in Diviſion being determined by thoſe of the Diviſor and Dividend, juſt in the ſame Manner as in Multiplication the Sign of the Product is determined by thoſe of the Multiplicator and Multiplicand; that is, if the Signs of the Diviſor and Dividend be both *alike*, whether they be both $+$ or $-$, the Quotient will be $+$; otherwiſe it will be negative. Thus if the Quantity $-12ab$ is divided by $-3a$, the Quotient will be $+4b$; which I thus demonſtrate.

In all Diviſions whatever, the Quotient ought to be ſuch a Quantity as, being multiplied by the Diviſor, will be equal to the Dividend; therefore to enquire for the Quotient in our Caſe, is nothing elſe, but to enquire what Number or Quantity multiplied into $-3a$ the Diviſor, will produce $-12ab$ the Dividend. Firſt then I aſk, what Sign multiplied into $-$, the Sign of the Diviſor, will give $-$, the Sign of the Dividend; and the Anſwer is $+$; therefore $+$ is the Sign of the Quotient.

tient. In the next place I inquire what Number multiplied into 3, the Coefficient of the Divisor, will give 12, the Coefficient of the Dividend; and the Answer is 4; therefore 4 is the Coefficient of the Quotient. Lastly, I enquire what Letter multiplied into the Letter of the Divisor will produce *ab* the Denominator, or literal Part of the Dividend; and the Answer is *b*; therefore *b* is the Letter of the Quotient. And thus at last we have the Quotient, which is $4b$; and this Way of reasoning will carry the Learner thro' all the other Cases.

EXAMPLES.

Divide	$+mn$	$-mn$	$+mn$	$-mn$	$ax+ay$
By	$+m$	$-m$	$-m$	$+m$	a
Quotient	$+n$	$+n$	$-n$	$-n$	$x+y$

For $n\times m=mn$, and $\overline{x+y}\times a=ax+ay$. Also $\frac{bx+mx}{m+b}$ is $=x$, and $\frac{ax-ca+nx-cn}{x-c}$ is $=a+n$.

SCHOLIUM I.

If a *Quantity* is found to be a common *Multiplier* in both, it may *be expunged from both*.

EXAMPLES.

Divide	ab	$ab+ad$	$ab+abc+abx$	$bcd+cdr$
By	db	acd	$abd+abf$	$cgd-cmd$
Quotient	$\frac{a}{d}$	$\frac{b+d}{cd}$	$\frac{1+c+x}{d+f}$	$\frac{b+r}{g-m}$

The Division of compound *Algebraic Quantities* is performed, first by ranging the several Members, both of the Divisor and Dividend, according to the Dimensions of some *Letter* common to them both, and then proceeding as in vulgar Arithmetic.

EXAMPLES

EXAMPLES.

$$
\begin{array}{l}
2x+3a)16x^4+144ax^3+476a^2x^2+684a^3x+360a^4(8x^3+60ax^2+148a^2x \cdot : 120a^3 \\
\qquad\quad \underline{16x^4+24ax^3} \\
\qquad\qquad\qquad 120ax^3+476a^2x^2 \\
\qquad\qquad\qquad \underline{120ax^3+180a^2x^2} \\
\qquad\qquad\qquad\qquad\quad 296a^2x^2+684a^3x \\
\qquad\qquad\qquad\qquad\quad \underline{296a^2x^2+444a^3x} \\
\qquad\qquad\qquad\qquad\qquad\qquad 240a^3x+360a^4 \\
\qquad\qquad\qquad\qquad\qquad\qquad \underline{240a^3x+360a^4} \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad \cdot \qquad \cdot
\end{array}
$$

$$
\begin{array}{l}
2x+6a)16x^4+144ax^3+476a^2x^2+684a^3x+360a^4(8x^3+48ax^2+94a^2x+60a^3. \\
\qquad\quad \underline{16x^4+48ax^3} \\
\qquad\qquad\qquad 96ax^3+476a^2x^2 \\
\qquad\qquad\qquad \underline{96ax^3+288a^2x^2} \\
\qquad\qquad\qquad\qquad\quad 188a^2x^2+684a^3x \\
\qquad\qquad\qquad\qquad\quad \underline{188a^2x^2+564a^3x} \\
\qquad\qquad\qquad\qquad\qquad\qquad 120a^3x+360a^4 \\
\qquad\qquad\qquad\qquad\qquad\qquad \underline{120a^3x+360a^4} \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad \cdot \qquad \cdot
\end{array}
$$

$3x+4a$

$$3x+4a)81x^4-108ax^3-495a^2x^2+354a^3x+840a^4(27x^3-72ax^2-69a^2x+210a^3.$$

$$\begin{array}{l}
\underline{81x^4+108ax^3} \\
\quad -216ax^3-495a^2x^2 \\
\quad \underline{-216ax^3-288a^2x^2} \\
\qquad -207a^2x^2+354a^3x \\
\qquad \underline{-207a^2x^2-276a^3x} \\
\qquad\quad +630a^3x+840a^4 \\
\qquad\quad \underline{+630a^3x+840a^4} \\
\qquad\qquad \cdot \quad \cdot
\end{array}$$

$$3x-7a)81x^4-108ax^3-495a^2x^2+354a^3x+840a^4(27x^3+27ax^2-102a^2x-120a^3$$

$$\begin{array}{l}
\underline{81x^4-189ax^3} \\
\quad 81ax^3-495a^2x^2 \\
\quad \underline{81ax^3-189a^2x^2} \\
\qquad -306a^2x^2+354a^3x \\
\qquad \underline{-306a^2x^2+714a^3x} \\
\qquad\quad -360a^3x+840a^4 \\
\qquad\quad \underline{-360a^3x+840a^4} \\
\qquad\qquad \cdot \quad \cdot
\end{array}$$

$$2x-3a)\,16x^4 * -72a^2x^2 * +81a^4\,(8x^3+12ax^2-18a^2x-27a^3$$

$$\underline{16x^4-24ax^3}$$

$$+24ax^3-72a^2x^2$$

$$\underline{+24ax^3-36a^2x^2}$$

$$-36a^2x^2 *$$

$$\underline{-36a^2x^2+54a^3x}$$

$$-54a^3x+81a^4$$

$$\underline{-54a^3x+81a^4}$$

$$\cdot \quad \cdot$$

$$3x+4a)\,81x^4 * * * -256a^4\,(27x^3-36ax^2+48a^2x-64a^3$$

$$\underline{81x^4+108ax^3}$$

$$-108ax^3 *$$

$$\underline{-108ax^3-144a^2x^2}$$

$$+144a^2x^2 *$$

$$\underline{+144a^2x^2+192a^3x}$$

$$-192a^3x-256a^4$$

$$\underline{-192a^3x-256a^4}$$

$$\cdot \quad \cdot$$

$2a^3$—

$$2a^3-3a^2+a)\,6a^{11}-33a^{10}+55a^9-66a^7+66a^5-33a^3+5a\,(3a^8-12a^7+8a^6+18a^5-10a^4-24a^3+2a^2+15a+5.$$

$$\begin{array}{l}
\underline{6a^{11}9-a^{10}+3a^9} \\
\quad -24a^{10}+52a^9\ * \\
\quad \underline{-24a^{10}+36a^9-12a^8} \\
\qquad\quad +16a^9+12a^8-66a^7 \\
\qquad\quad \underline{16a^9-24a^8+8a^7} \\
\qquad\qquad\quad 36a^8-74a^7 \\
\qquad\qquad\quad \underline{36a^8-54a^7+18a^6} \\
\qquad\qquad\qquad -20a^7-18a^6+66a^5 \\
\qquad\qquad\qquad \underline{-20a^7+30a^6-10a^5} \\
\qquad\qquad\qquad\qquad -48a^6-76a^5\ * \\
\qquad\qquad\qquad\qquad \underline{-48a^6+72a^5-24a^4} \\
\qquad\qquad\qquad\qquad\qquad 4a^5+24a^4-33a^3 \\
\qquad\qquad\qquad\qquad\qquad \underline{4a^5-6a^4+2a^3} \\
\qquad\qquad\qquad\qquad\qquad\quad +30a^4-35a^3\ * \\
\qquad\qquad\qquad\qquad\qquad\quad \underline{30a^4-45a^3+15a^2} \\
\qquad\qquad\qquad\qquad\qquad\qquad +10a^3-15a^2+5a \\
\qquad\qquad\qquad\qquad\qquad\qquad \underline{10a^3-15a^2+5a} \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad .\quad .\quad .
\end{array}$$

SCHO-

SCHOLIUM II.

When the *Divisor* is not an even Part of the *Dividend*, you may carry on the Operation by annexing to the *Quotient* the Remainder set over the *Divisor*, with a Line drawn between them, in order to terminate it; or else carryed on in an *infinite Series*.

EXAMPLES.

$$x+y)zz\left(\frac{zz}{x}-\frac{z^2y}{x^2}+\frac{z^2y^2}{x^3}-\frac{z^2y^3}{x^4}\right., \text{ \&c. } in\ infinitum.$$

$$zz+\frac{zzy}{x}$$

$$-\frac{zzy}{x}+0.$$

$$-\frac{zzy}{x}-\frac{zzy^2}{x^2}$$

$$+\frac{z^2y^2}{x^2}+0$$

$$+\frac{z^2y^2}{x^2}+\frac{z^2y^3}{x^3}$$

$$-\frac{z^2y^3}{x^3}+0$$

$$-\frac{z^2y^3}{x^3}-\frac{z^2y^4}{x^4}$$

$$+\frac{z^2y^4}{x^4}$$

For $\frac{\overline{x+y}}{1}\times\frac{zz}{x}=\frac{z^2x+z^2y}{x}=z^2+\frac{z^2y}{x}$.

$$\frac{\overline{x+y}}{1}-\frac{z^2y}{x^2}=-\frac{z^2xy-z^2y^2}{x^2}=-\frac{z^2y}{x}-\frac{z^2y^2}{x^2}.$$

$$\frac{\overline{x+y}}{1}\times\frac{z^2y^2}{x^3}=\frac{z^2xy^2+z^2y^3}{x^3}=\frac{z^2y^2}{x^2}+\frac{z^2y^3}{x^3}.$$

$$\frac{\overline{x+y}}{1}\times-\frac{z^2y^3}{x^4}=\frac{-xz^2y^3-z^2y^4}{x^4}=\frac{-z^2y^3}{x^3}-\frac{z^2y^4}{x^4}.$$

SCHOLIUM III.

The Law of Continuation is manifest, and may be carried on to any Degree of Exactness.

$$y+x)zz\left(\frac{zz}{y}-\frac{z^2x}{y^2}+\frac{z^2x^2}{y^3}-\frac{z^2x^3}{y^4}\right., \text{ \&c. } \textit{in infinitum.}$$

$$zz+\frac{zzx}{y}$$

$$\frac{-zzx}{y}+0$$

$$\frac{-zzx}{y}-\frac{zzx^2}{y^2}$$

$$\frac{+z^2x^2}{y^2}+0$$

$$\frac{z^2x^2}{y^2}+\frac{z^2x^3}{y^3}$$

$$\frac{-z^2x^3}{y^3}+0$$

$$\frac{-z^2x^3}{y^3}+\frac{z^2x^4}{y^4}$$

$$\frac{-z^2x^4}{y^4} \text{ \&c.}$$

For $\frac{\overline{y+x}}{1}\times\frac{zz}{y}=\frac{z^2y+z^2x}{y}=z^2+\frac{z^2x}{y}$

$$\frac{\overline{y+x}}{1}\times-\frac{z^2x}{y^2}=-\frac{z^2xy-z^2x^2}{y^2}=-\frac{z^2x}{y}-\frac{z^2x^2}{y^2}$$

$$\frac{\overline{y+x}}{1}\times\frac{z^2x^2}{y^3}=\frac{z^2x^2y+z^2x^3}{y^3}=\frac{z^2x^2}{y^2}+\frac{z^2x^3}{y^3}$$

$$\frac{\overline{y+x}}{1}\times-\frac{z^2x^3}{y^4}=\frac{z^2x^3y-z^2x^4}{y^4}=\frac{-z^2x^3}{y^3}-\frac{z^2x^4}{y^4}$$

$1+xx)1(1-x^2+x^4-x^6+x^8$ &c. *in infinitum.*

$$1+xx$$
$$-xx+0$$
$$-xx-x^4$$
$$+x^4+0$$
$$+x^4+x^6$$
$$-x^6+0$$
$$-x^6-x^8$$
$$+x^8+0$$
$$+x^8+x^{10}$$
$$-x^{10} \text{ \&c.}$$

Whence the Law of the Continuation is evident.

$xx+1)1\left(\frac{1}{xx}-\frac{1}{x^4}+\frac{1}{x^6}-\frac{1}{x^8},\right.$ &c. *in infinitum.*

$$1+\frac{1}{xx}$$
$$-\frac{1}{xx}+0$$
$$-\frac{1}{xx}-\frac{1}{x^4}$$
$$+\frac{1}{x^4}+0, \text{ \&c.}$$

For $\frac{\overline{xx+1}}{1}\times\frac{1}{xx}=\frac{xx+1}{xx}=1+\frac{1}{xx}.$

And $\frac{\overline{xx+1}}{1}\times-\frac{1}{x^4}=-\frac{xx+1}{x^4}=-\frac{1}{xx}-\frac{1}{x^4}$

&c.

$$xx-yy)\,xxy\,\Big(y+\frac{y^3}{xx}+\frac{y^5}{x^4}+\frac{y^7}{x^6}+\frac{y^9}{x^8}\ \text{\&c. } \textit{in infinitum.}$$

$$\begin{array}{l}
\underline{xxy-y^3} \\
\quad +y^3+0 \\
\quad \underline{y^3-\frac{y^5}{x^2}} \\
\qquad +\frac{y^5}{x^2}+0 \\
\qquad \underline{\frac{y^5}{x^2}-\frac{y^7}{x^4}} \\
\qquad\quad +\frac{y^7}{x^4},\ \text{\&c.}
\end{array}$$

$$xx-yy)\,xxxz\,\Big(xz+\frac{y^2z}{x}+\frac{y^4z}{x^3}+\frac{y^6z}{x^5},\ \text{\&c. } \textit{in infinitum.}$$

$$\begin{array}{l}
\underline{xxxz-xzyy} \\
\quad +xzyy+0 \\
\quad \underline{xzyy-\frac{y^4z}{x}} \\
\qquad +\frac{y^4z}{x}+0 \\
\qquad \underline{\frac{y^4z}{x}-\frac{y^6z}{x^3}} \\
\qquad\quad +\frac{y^6z}{x^3}+0 \\
\qquad\quad \underline{\frac{y^6z}{x^3}-\frac{y^8z}{x^5}} \\
\qquad\qquad +\frac{y^8z}{x^5} \\
\qquad\qquad \text{\&c.}
\end{array}$$

Note, It is proper to obſerve that the greateſt Term of the Series muſt ſtand firſt; otherwiſe the Series will not be true, without taking in the Remainder for the Numerator of a Fraction whoſe Denominator is the Diviſor. Theſe ſix laſt Examples ſhow how to throw an Expreſſion into an *infinite Series*, which Beginners may omit, till they are better acquainted with Fractions.

INVOLUTION of QUANTITIES.

DEFINITION.

INvolution is the multiplying a Root or Number into or by itself, which when done once is called the second Power, or a Square; when that Product is again multiplied by the Root, or original Number, it is called the third Power, or the Cube, &c. *Therefore, I say, a Quantity multiplied any number of Times is said to be* involved, *the* Products *arising from thence are called* Powers, *and the* Quantity *so multiplied a* Root.

Thus $\begin{cases} a\times a = aa, \text{ or } a^2. \\ a\times a\times a = aaa, \text{ or } a^3. \\ a\times a\times a\times a = aaaa, \text{ or } a^4. \\ a\times a\times a\times a\times a = aaaaa, \text{ or } a^5. \end{cases}$

The usual Character or Sign for *Involution* is ⊕.

Quantities, compounded of several Terms, are involved by an actual Multiplication of all their Parts.

Thus to raise $m+n$ to the fourth Power. *See the Operation.*

First Power $= m+n =$ Root.

$$\begin{array}{ll}
\text{First Power} & = m+n = \text{Root.} \\
& \underline{m+n} \\
& m^2+mn \\
& \underline{\quad +mn+nn} \\
\text{Second Power} & = m^2+2mn+nn = \text{Square, or } \overline{m+n}|^2 \\
& \underline{m+n} \\
& m^3+2m^2n+mnn \\
& \underline{\quad +m^2n+2mn^2+n^3} \\
\text{Third Power} & = m^3+3m^2n+3mn^2+n^3 = \text{Cube, or } \overline{m+n}|^3. \\
& \underline{m+n} \\
& m^4+3m^3n+3m^2n^2+mn^3 \\
& \underline{\quad +m^3n+3m^2n^2+3mn^3+n^4} \\
\text{Fourth Power} & = m^4+4m^3n+6m^2n^2+4mn^3+n^4 = \overline{m+n}|^4
\end{array}$$

Note,

Note, $\overline{m+n}|^2$ denotes the Square of $m+n$. $\overline{m+n}|^3$ denotes $m+n$ cub'd, or rais'd to the third Power; and is no more than $m^3+3m^2n+3mn^2+n^3$ expressed in shorter Terms; and so of any other.

This Method of Notation is now become vastly useful in literal Computations, as solving Problems, &c.

So likewise if it be required to involve $m-n$ to the fourth Power, the Process will stand thus:

First Power $= m-n =$ Root

$$\begin{array}{l} m-n \\ \hline mm-mn \\ \quad -mn+nn \\ \hline \end{array}$$

Second Power $= m^2-2mn+n^2 =$ Square, or $\overline{m-n}|^2$.

$$\begin{array}{l} m-n \\ \hline m^3-2m^2n+mn^2 \\ \quad -m^2n+2mn^2-n^3 \\ \hline \end{array}$$

Third Power $= m^3-3m^2n+3mn^2-n^3 =$ Cube, or $\overline{m-n}|^3$

$$\begin{array}{l} m-n \\ \hline m^4-3m^3n+3m^2n^2-mn^3 \\ \quad -m^3n+3m^2n^2-3mn^3+n^4 \\ \hline \end{array}$$

Fourth Power $= m^4-4m^3n+6m^2n^2-4mn^3+n^4 = \overline{m-n}|^4$ the Power required, and so of any other.

The Method of proceeding is the same in the generating of higher *Powers* from any given *Root*, whether *Binomial* or *Multinomial*. As

If $m+n+a-r$ were to be squared.

Thus

First Power $= m+n+a-r =$ Root

$$\begin{array}{l} m+n+a-r \\ \hline m^2+mn+am-rm \\ \quad +mn+nn+an-rn \\ \qquad +am+an+aa-ar \\ \qquad\quad -rm-rn-ar+rr \\ \hline \end{array}$$

Square $= m^2+2mn+2am+n^2+2an-2rm-2rn+aa-2ar+rr$

Besides

Besides this Method of forming Powers by multiplying the Quantities at length, there is another Rule which performs the Work much shorter, especially in Binomial Quantities; as if $a+e$ was to be involved to the 7th Power.

RULE.

The Power required must be made to consist of one Term more, than is the Index of the Power required, whereof the first and last are pure Powers, the first of a, the last of e, each involved to the Power required; then all the intermediate Terms are to be made up of both a and e; and in these several Terms the Index of a decreases, and that of e increases gradually, so that the Sum of their Indices in any Term is equal to the Index of the required Power.

And for finding the Coefficients, or *Unciae*, of the several Terms, the first is always 1, the second is the Index of the Power required. And, in general, if the *Uncia* of any Term be multiplied by the Index of the leading Quantity (a), and divided by the Number of Terms to that Place, it gives the *Uncia* of the next following Term.

Thus, to find the 7th Power of $a+e$, the Terms without the *Unciae* will be a^7, a^6e, a^5e^2, a^4e^3, a^3e^4, a^2e^5, ae^6, e^7; and the particular *Unciæ*, or Coefficients, by the Rule are 1, 7, $\frac{7\times6}{2}=21$, $\frac{21\times5}{3}=35$, $\frac{35\times4}{4}=35$, $\frac{35\times3}{5}=21$, $\frac{21\times2}{6}=7$, and $\frac{7\times1}{7}=1$. Therefore $\overline{a+e}|^7$ $=a^7+7a^6e+21a^5e^2+35a^4e^3+35a^3e^4+21a^2e^5+7ae^6+e^7$.

And the same Rule holds when the following Quantity (e) is negative, only the 2d, 4th, 6th, &c. Terms are negative. Thus

$\overline{a-b}|^7=a^7-7a^6b+21a^5b^2-35a^4b^3+35a^3b^4-21a^2b^5+7ab^6-b^7$.

EVO-

EVOLUTION of QUANTITIES.

DEFINITION.

EVolution is the extracting Roots out of any Power given, and therefore is called the Analysis *of Powers, and is just the Reverse of Involution.*

The usual Sign, or Character, for Evolution is $\surd$.

Thus the Square Root of aa is a
of $m^2x^2y^2$ is mxy
of $m^4x^4y^4$ is $m^2x^2y^2$
of mxy is $\sqrt{mxy}$, or $\overline{mxy}|^{\frac{1}{2}}$

Note, $\sqrt{mxy}$, $\overline{mxy}|^{\frac{1}{2}}$ are both the same Expression, the former being the old, the latter the new Way of Notation for the Square Root of mxy, &c.

Also the Cube Root of a^3 is a
of x^6y^6 is x^2y^2
of x^3y^3 is xy
of xy is $\overline{xy}|^{\frac{1}{3}}$ or $\sqrt[3]{xy}$

The Square Root of a is $a^{\frac{1}{2}}$, or $\sqrt{a}$.
of $a+x$ is $\sqrt{a+x}$, or $\overline{a+x}|^{\frac{1}{2}}$
of $x+\sqrt{mn}+y$ is $\overline{x+\sqrt{mn}+y}|^{\frac{1}{2}}$, or $\sqrt{x+\sqrt{mn}+y}$

Also the Cube Root of x is $x^{\frac{1}{3}}$, or $\sqrt[3]{x}$
of $a+x$ is $\overline{a+x}|^{\frac{1}{3}}$, or $\sqrt[3]{a+x}$
of $x+\sqrt{mn}+y$ is $\overline{x+\sqrt{mn}+y}|^{\frac{1}{3}}$, or $\sqrt[3]{x+\sqrt{mn}+y}$

Likewise the Biquadrate Root of $\Big\}$ a^4 is a
of x^4y^4 xy
of x is $x^{\frac{1}{4}}$, or $\sqrt[4]{x}$

of

of $a+x$ is $\overline{a+x}|^{\frac{1}{4}}$, or $\sqrt[4]{a+x}$.

of $\overline{xx+aa}|^{3}$ is $\overline{xx+aa}|^{\frac{3}{4}}$.

of $\overline{cx-gy+qq}|^{\frac{1}{3}}$ is $\overline{cx-gy+qq}|^{\frac{1}{12}}$.

The Square Root of $49xx$ is $7x$.

of $121x^4$ is $11x^2$.

of $11x^2$ is $\sqrt{11x^2}$, or $x\sqrt{11}$.

The Cube Root of $64y^3$ is $4y$.

of $722y^6$ is $9y^2$.

of $119m^3n^2$, is $\sqrt[3]{119m^3n^2}$, or $\overline{119m^3n^2}|^{\frac{1}{3}}$, or $m\sqrt[3]{119n^2}$.

Compound Quantities.

$aa+2ax+xx(a+x=$ Root.
aa
$2a)+2ax+xx$
$2ax+xx$
$\cdot\quad\cdot$

$xx-2ax+aa(x-a=$ the Root.
xx
$2x-a)-2ax+aa$
$-2ax+aa$
$\cdot\quad\cdot$

$xx-ax+\frac{1}{4}aa(x-\frac{1}{2}a=$ Root.
xx
$2x-\frac{1}{2}a)-ax+\frac{1}{4}aa$
$-ax+\frac{1}{4}aa$
$\cdot\quad\cdot$

Here the ſeveral Terms are placed in order, of which the given Quantity is compoſed according to the Demenſions of ſome Letters therein, as you ſhall think beſt; and then the Root of the firſt Term is found, and placed in the Quotient, as in compound Diviſion; and ſubtracting that Term from the given Quantity, and bringing down the firſt Term of the Remainder, and

 dividing

dividing by twice the first Term of the Quotient, or Root, just as the Extraction of the Square Root in whole Numbers; and so repeating the Operation, and dividing the first Term of the Remainder by the same Divisor as in the above Examples, and the Thing is done.

The Roots of higher Powers may sometimes be discovered thus: Extract the Root required out of all the simple Terms which will admit of such a Root; and connect them together with the Signs $+$ and $-$; which done, involve this Quantity to the same height as the given Power, and if it be the same throughout as the given compound Quantity, you have the Root; if it differ in the Signs, change some of the Signs, and involve it again till they agree.

EXAMPLE.

To extract the Cube Root of $a^3-6a^2b+12ab^2-8b^3$; here $\sqrt[3]{a^3}$ is $=a$, and $\sqrt[3]{-8b^3}$ is $=-2b$; therefore I take $a-2b$ for the Root, which being involved to the third Power, is $a^3-6a^2b+12ab^2-8b^3$; therefore $a-2b$ is the true Root. But if $a^3-6a^2b+12ab^2+8b^3$ was given, I try $a+2b$, and it produces $a^3+6a^2b+12ab^2+8b^3$, which differs from the Quantity given; which has no Root but what is surd, and must be expressed thus, $\sqrt[3]{a^3-6a^2b+12ab^2+8b^3}$; or thus, $\overline{a^3-6a^2b+12ab^2+8b^3}|^{\frac{1}{3}}$.

ALGE-

Algebraical Fractions.

A Fraction is a broken Number, expressing some Part or Parts of any thing consider'd as an Integer. It consists of two Quantities, placed one above another, with a Line drawn between them, as in Vulgar Fractions, thus:

Numerators / Denominators $\frac{2}{3}$, $\frac{a}{b}$, $\frac{2a-c}{3b}$, $\frac{5bx^2+m-y}{3-y+m}$, &c.

PROPOSITION I.

To Reduce fractional Quantities into a lower Denomination.

RULE.

Divide both Numerator and Denominator by their greatest common Divisor, or by any Quantity that will divide both of them, and the Quotients will be the new Fraction. See p. 18, 19. *Vulgar Fractions.*

Examples.

1. $\frac{mnx}{mxy}=\frac{n}{y}$. 2. $\frac{25z}{5xz+15az}=\frac{5}{x+3a}$.

3. $\frac{m^2-n^2}{m^2+2mn+n^2}=\frac{m-n}{m+n}$.

PROPOSITION II.

To reduce an Integer *into an* improper Fraction.

CASE I.

When there is no assigned Denominator, *then let the given* Integer *be a Numerator, and* Unit *its Denominator.* See Page 20.

Examples.

$x=\frac{x}{1}$; $m+n=\frac{m+n}{1}$; $rt=\frac{rt}{1}$; $x^2+y^2=\frac{x^2+y^2}{1}$.

CASE II.

When there is an assigned Denominator.

RULE.

Multiply the Integer by the assigned Denominator, the Product shall be the Numerator.

EXAMPLES.

1. Denom. assign'd a. Then $y=\frac{ya}{a}$; here the Integer is y.

2. Denom. assign'd x. Then $a+b=\frac{ax+bx}{x}$; here the Integer is $a+b$.

3. Denom. assign'd $m+n$. Then $x=\frac{mx+nx}{m+n}$; here the Integer is x.

For $y\times a=ya$; $\overline{a+b}\times x=ax+bx$; $x\times\overline{m+n}=mx+nx$.

PROPOSITION III.

TO reduce mixt Fractions *into* Improper Ones.

RULE.

Multiply the Integer by the Denominator of the Fraction, and add the Numerator to the Product, subscribing the same Denominotor. See Page 21.

EXAMPLES.

1. $x+\frac{mn}{b}=\frac{bx+mn}{b}$.

2. $a+b-\frac{zz}{mr}=\frac{amr+bmr-zz}{mr}$.

3. $x+y-cx+my+\frac{xz+yz}{r+c}=$

$$\frac{rx+ry-rcx+rmy+cx+cy-c^2x+cmy+xz+yz}{r+c}$$

PROPOSITION IV.

TO reduce an improper Fraction *into an* Integer, *or* mixt Fraction.

RULE.

The Numerator divided by the Denominator, the Quotient will be the mixt Fraction, or Integer required. See p. 22.

EXAMPLES.

1. $\frac{bx+mn}{b}=x+\frac{mn}{b}$.

2. $\frac{mx+nx}{m+n}=x$.

3. $\frac{amr+bmr-zz}{mr}=a+b-\frac{zz}{mr}$.

4. $\frac{rx+ry-rcx+rmy+cx+cy-c^2x+cmy+xz+yz}{r+c}=$

$x+y-cx+my+\frac{xz+yz}{r+c}$.

PROPOSITION V.

TO reduce Fractions *of different* Denominators *into their Equivalents, which shall have the same* Denominator.

RULE.

Multiply all the Denominators continually for a common Denominator, and each Numerator continually by the others Denominators for new Numerators, and the Products are the respective Numerators. See Rule, Page 24.

EXAMPLES.

1. $\frac{a}{b}$, $\frac{c}{d}$ make $\frac{ad}{bd}$, $\frac{bc}{bd}$.

For $\left\{ \frac{a\times d}{b\times d}=\frac{ad}{bd}, \text{ and } \frac{c\times b}{d\times b}=\frac{bc}{bd}. \right.$

2. $\frac{y}{x}$, $\frac{z}{m}$, $\frac{ab}{c}$, when reduced, make $\frac{cmy}{cmx}$, $\frac{czx}{cmx}$, $\frac{abmx}{cmx}$.

$$\text{For}\left\{\begin{array}{l} \dfrac{y\times m\times c}{x\times m\times c}=\dfrac{cmy}{cmx}\left(=\dfrac{y}{x}.\right. \\ \dfrac{z\times x\times c}{m\times x\times c}=\dfrac{czx}{mxc}\left(=\dfrac{z}{m}.\right. \\ \dfrac{ab\times x\times m}{c\times x\times m}=\dfrac{abmx}{cxm}\left(=\dfrac{ab}{c}.\right. \end{array}\right\}$$

PRO-

Addition of Fractions.

PROPOSITION VI.

To add a Fraction to another, or to ſubtract one from another.

Theſe are managed as Vulgar Fractions. *See p.* 33—45. *For when theſe are reduced to a common Denominator, the Sum or Difference of the Numerators, ſet over the common Denominator, will be the Sum, or Difference, of the given Fractions.*

Examples in *Addition*.

1. $\frac{a}{b}$, add to $\frac{c}{d}$, is $= \frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$; *by Prop.* 5.

2. $\frac{xx}{m}$, added to $\frac{yy}{m}$, is $= \frac{xx}{m} + \frac{yy}{m} = \frac{xx+yy}{m}$.

3. $\frac{xx+xy}{x-y}$, add to x, $= \frac{xx+xy+xx-xy}{x-y} = \frac{2xx}{x-y}$.

4. $\frac{xx}{a}$ added to $\frac{yy}{b}$, added to $\frac{zz}{c}$, is $= \frac{bcxx+acyy+abzz}{abc}$

$$\text{For } \left\{ \begin{array}{l} \frac{xx\times b\times c}{a\times b\times c} = \frac{bcxx}{abc}. \\ \frac{yy\times a\times c}{b\times a\times c} = \frac{acyy}{abc}. \\ \frac{zz\times a\times b}{c\times a\times b} = \frac{abzz}{abc}. \end{array} \right\} \textit{ by Prop. } 5.$$

Therefore $\frac{bcxx}{abc} + \frac{acyy}{abc} + \frac{abzz}{abc}$ is $= \frac{bcxx+acyy+abzz}{abc}$; by this *Propoſition*.

5. $\frac{aa}{m} + \frac{bb}{n} + \frac{cc}{d} + \frac{zz}{xx}$ is $=$

and $\frac{xx+b^2mdx^2+c^2mnx^2+z^2mnd}{mndxx}$

Sub-

Subtraction of Fractions.

EXAMPLES in *Subtraction.*

1. From $\frac{a}{b}$ take $\frac{c}{d}$, the Remainder is $\frac{ad-bc}{bd}$.

$$\left.\begin{array}{l}\text{For } \frac{a\times d}{b\times d}=\frac{ad}{bd}\\ \text{And } \frac{c\times b}{d\times b}=\frac{bc}{bd}\end{array}\right\}\ \textit{by Prop. 5.}$$

Therefore $\frac{ad}{bd}-\frac{bc}{bd}$ is $=\frac{ad-bc}{bd}$; by this Proposition.

2. $\frac{xx}{m}-\frac{yy}{m}=\frac{xx-yy}{m}$; as is evident.

3. From $\frac{a^4+dx^3}{abd}$ take $\frac{x^3}{ab}$, the Remainder is $=\frac{aaa}{bd}$.

For $\frac{a^4+dx^3}{abd}-\frac{x^3}{ab}=\left(\frac{a^5b+abdx^3-abdx^3}{a^2b^2d}=\right)\frac{aaa}{bd}$.

4. From $\frac{2xy+y^2}{x+y}$ take y, the Remainder is $\frac{xy}{x+y}$.

For $\frac{2xy+y^2}{x+y}-y=\left(\frac{2xy+y^2-xy-y^2}{x+y}=\right)\frac{xy}{x+y}$.

5. From $\frac{a^3+n^3}{cx-xx}$ take $\frac{n^4}{a^2c-a^2x}$, the Remainder is $=\frac{a^5+a^2n^3-n^4x}{a^2cx-a^2x^2}$.

Mul-

PROPOSITION VII.

Multiplication of Fractions.

RULE.

FIRST reduce whole or mixt Quantities into improper Fractions, and then multiply the Numerators *for a new* Numerator, *and the* Denominators *for a new* Denominator. *See Page* 46, 47, &c.

EXAMPLES.

1. $\frac{a}{b}$ multiplied by $\frac{c}{d}$ is $= \frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.

2. $\frac{mx}{cy} \times \frac{dy}{cz} = \frac{mdxy}{ccyz}$.

3. $\frac{bx}{c}$ multiplied by x is $\frac{bxx}{c}$; for $\frac{bx}{c} \times \frac{x}{1} = \frac{bxx}{c}$.

4. $\frac{ax}{m} \times a+x$ is $= \frac{ax}{m} \times \frac{\overline{a+x}}{1} = \frac{aax+axx}{m}$.

5. $\frac{x^3}{ax+aa}$ multiplied by $\overline{x+a}$ is $= \frac{x^3}{ax+aa} \times \frac{x+a}{1} =$ $\frac{x^4+ax^3}{ax+aa} = \frac{x^3}{a}$.

6. $x + \frac{aa}{x-a}$ multiplied by $x-a$ is $= \frac{\overline{xx-ax+aa}}{x-a} \times$ $\overline{x-a} = xx-ax+aa$.

7. $x + \frac{aa}{x-a} \times x - 2a + \frac{aa}{x}$ is $= \frac{x^2-ax+aa}{x-a} \times$ $\frac{xx-2ax+aa}{x} = \frac{x^4-3ax^3+4a^2x^2-3a^3x+a^4}{x-ax} = x^2 -$ $2ax + 2a^2 - \frac{a^3}{x}$.

For $\frac{a^3}{x} \times \overline{xx-ax} = \frac{a^3xx-a^4x}{x} = a^3x - a^4$.

If any Fraction be to be multiplied by the Denominator, the Numerator is the Product.

PRO-

PROPOSITION VIII.

Division of Algebraic Fractions.

IS performed just in the same Way as Vulgar Fractions; invert the Divisor, and multiply them by the last Rule. *See page* 52, 53, *&c.*

EXAMPLES.

1. $\frac{a}{b}$ divided by $\frac{x}{y}$, quotes $\frac{ay}{bx}$.

   For $\frac{y}{x} \times \frac{a}{b} = \frac{ay}{bx}$.

2. $\frac{axy}{bcm}$ divided by $\frac{r}{s}$ quotes $\frac{asxy}{bcmr}$.

   For $\frac{s}{r} \times \frac{axy}{bcm} = \frac{asxy}{bcmr}$.

3. $\frac{a+3}{5}$ divided by $\frac{20x}{3}$ gives $\frac{3a+9}{100x}$.

   For $\frac{3}{20x} \times \frac{a+3}{5} = \frac{3a+9}{100x}$.

4. $\frac{mx+21}{17}$ divided by $\frac{11yz}{12}$ gives $\frac{12mx+252}{187yz}$.

   For $\frac{12}{11yz} \times \frac{mx+21}{17} = \frac{12mx+252}{187yz}$.

5. $\frac{y^3}{aa}$ divided by x gives $\frac{y^3}{a^2x}$.

   For $\frac{1}{x} \times \frac{y^3}{a^2} = \frac{y^3}{a^2x}$.

6. $x \div \frac{y^3}{a^2} = \frac{a^2}{y^3}\Big)\frac{x}{1}\Big(= \frac{a^2x}{y^3} = \frac{a^2}{y^3} \times \frac{x}{1} = \frac{a^2x}{y^3}$.

7. $x-y \div \frac{x-y}{a} = \frac{a}{x-y}\Big)\frac{x-y}{1}\Big(= \frac{ax-ay}{x-y} = a$.

8. x^2+

8. $\frac{x^2+2ax+a^2-bx-ab}{ax+bx-ab-bb} \div \frac{x+a-b}{a+b} =$

$\frac{a+b}{x+a-b}\Big)\frac{x^2+2ax+a^2-bx-ab}{ax+bx-ab-bb}\Big(= \frac{x+a}{x-b}.$

The Demonſtration to the above may be had thus; an Inſtance of which take Example firſt; where it is ſaid that $\frac{a}{b} \div \frac{x}{y}$ gives $\frac{ay}{bx}$.

For $\frac{a}{b}, \frac{x}{y} = \frac{ay}{by}, \frac{bx}{by}$; by *Prop.* 5.

But $\frac{ay}{by} : \frac{bx}{by} :: ay : bx.$

That is, $\frac{a}{b} : \frac{x}{y} :: ay : bx$; by equal Diviſion.

Therefore $\frac{a}{b} \div \frac{x}{y} = \frac{ay}{bx}$.

When the Fractions have a common Denominator, divide one Numerator by the other.

Becauſe Fractions, having the ſame Denominators, are as their Numerators; by *Cor.* 3. *page* 14.

Of Proportional Quantities.

Ratio, or *Proportion*, is the Relation which one Quantity has to another in reſpect of Magnitude; of theſe Quantities the former is called the *Antecedent*, and the latter the *Conſequent*.

The *Quantity* of any *Ratio* is the Quotient of the Antecedent divided by the Conſequent, and expreſſes how many fold the one is of the other. See *Def.* 31. *page* 6.

Thus if A, B be two Quantities, A the Antecedent, and B the Conſequent, the Ratio of A to B is expreſſed by $\frac{A}{B}$; as if A be 4, and B 2, then the Ratio of 4 to 2 is $\frac{4}{2}$, or 2, that is, double.

Quantities are ſaid to be *proportional*, or to have the *ſame Proportion* to one another, when one Antecedent con-

contains its Consequent as often as any other Antecedent contains its Consequent. Thus if $\frac{A}{B} = \frac{C}{D}$, then the Quantities A, B, C, D, are called *Proportionals*, and are said to be *geometrically proportional*; and are written thus, A : B :: C : D. In Numbers, if $\frac{8}{4} = \frac{6}{3}$, then 8 is to 4 as 6 is to 3, or 8 : 4 :: 6 : 3.

If there be three or more Quantities A, B, C, D, &c. and if A : B :: B : C :: C : D, &c. then the Quantities A, B, C, D, &c. are said to be in *continual Proportion*, or in *Geometrical Progression*; and are written thus, A, B, C, D $\div\!\div$.

PROPOSITION IX.

IF four Quantities A, B, C, D, are proportional, the Rectangle of the Means is equal to the Rectangle of the Extremes; BC=AD.

For by Supposition $\frac{A}{B} = \frac{C}{D}$; then multiplying both by B (by *Ax.* 2.) $\frac{A \times B}{B}$, or $A = \frac{CB}{D}$; and again multiplying both by D, $A \times D = \frac{CB \times D}{D} = CB$.

COROLLARY I.

Hence it follows that if the Product of two Quantities, AD, be equal to the Product of two other Quantities, BC, these four Quantities are proportional, A : B :: C : D.

COROLLARY II.

Hence if three Quantities are in continual Proportion, the Square of the Mean is equal to the Rectangle of the Extremes; for in this Case B=C, and $AD = BC = B^2$ or C^2.

COROLLARY III.

Hence also, if the Rectangle of two Quantities be equal

equal to the Square of a third; these three Quantities are in continual Proportion.

Note, The Learner will find this Proposition to be of great Service in bringing Problems to Equations.

PROBLEM I.

TO turn proportional Quantities into Equations.

RULE.

Make the Product of the Means equal to the Product of the Extremes.

EXAMPLES.

1. If $a : b :: c : d$, then $ad = bc$.
2. If $a : b :: b : c$, then $bb = ac$.
3. If $c+d : b :: c : a$, then $bc = ca + da$.
4. If $2b+y : a+2e :: a : y$, then $2by + yy = aa + 2ae$

PROBLEM II.

TO turn Equations into Analogies or Proportions.

RULE.

Divide each Side of the Equation into two such Factors as being multiplied together will produce that side. Then make the two Factors of one side of the Equation the two Means, and the two Factors, on the other side, the two Extremes.

EXAMPLES.

1. If $ad = bc$, then $a : b :: c : d$.
2. If $ee = pq$, then $p : e :: e : q$.
3. If $bc = ca + da$, then $c+d : b :: c : a$.
4. If $aa + 2ae = 2by + yy$, then $a : \overline{y} :: 2b+y : \overline{a+2e}$.
5. If $bc + bd = da + g$, then $b : \sqrt{da+g} :: \sqrt{da+g} : c+d$; or thus, $b : 1 :: da+g : c+d$.

PROPOSITION X.

Reduction of Simple Equations.

AN Equation in Algebra is that Equality which always exiſts between two different Expreſſions, as between one Number, or Quantity, and one; ſeveral, and one; ſeveral and ſeveral, or between their Sums, Differences, Products, Quotients, Powers and Roots, either all expreſſed particularly by the common numerical Characters, or univerſally by the Letters of the Alphabet, or by both theſe together, and known by this Mark $=$. The Uſe of ſuch Equations is for repreſenting more conveniently and more diſtinctly the Conditions of Problems, when tranſlated out of common Language into that of *Algebra*. As, for Example, let it be ſuppoſed to find a Number with the following Property, *viz.* that $\frac{2}{3}$ of it, with 4 over, may amount to the ſame as $\frac{7}{12}$ with 9 over. Here putting x for the unknown Quantity, the Condition of this Problem (when tranſlated out of common Language, into that of Algebra) will be repreſented by the following Equation, *viz.* $\frac{2x}{3}+4=\frac{7x}{12}+9$. Now ſince in this Equation, as well as almoſt all others, ariſing immediately from the Conditions of Problems themſelves, the unknown Quantity x, muſt be ſo ordered, that itſelf alone poſſeſſing one ſide of the Equation, muſt be found equal to ſuch as are entirely known on the other, that is, in the preſent Caſe, to determine the Value of the unknown Quantity x, is what is commonly called the *Reſolution of an Equation*; for the effecting whereof ſeveral Precepts, Axioms and Proceſſes are required, ſome whereof, to wit, ſuch as moſt fre-

frequently occur, we ſhall here put down, the reſt we ſhall take notice of occaſionally as they offer themſelves.

1. If any Quantity be taken from one ſide of an Equation, and be placed on the other with a *contrary* Sign, which is commonly called *Tranſpoſition*, the two Sides of the Equation will be equal to one another. Thus $12-4=8$; tranſpoſe -4, and you will have $12=8+4=12$; or $x-a=b$; tranſpoſe $-a$, and you will have $x=b+a$.

2. If when an Equation is to be reſolved, Fractions are found on one or both Sides, it muſt be freed from them by multiplying the whole Equation into the *Denominators* of thoſe Fractions ſucceſſively. Thus if $\frac{2x}{3}+4=\frac{7x}{12}+9$, multiply both ſides by 3, and you will have $2x+12=\frac{21x}{12}+27$; multiply again by 12, and you will have $24x+144=21x+324$.

3. If, after the Equation is thus reduced to integral Terms, the unknown Quantity be found on both Sides the Equation (which in moſt Caſes happens ſo) let it be brought by *Tranſpoſition* to one and the ſame Side, *viz.* to that Side which, after *Reduction*, will exhibit it *affirmative*. Thus if $24x+144=21x+324$, tranſpoſe $21x$, and you will have $24x-21x+144=324$, or rather $3x+144=324$.

After this, if any looſe known Quantities be found on the ſame Side with the unknown, let them alſo be tranſpoſed to the other Side of the Equation; as we have here 144, joined to the unknown Quantity above, which tranſpoſed is $3x=324-144$, or rather $3x=180$.

4. If now the unknown Quantity has any *Coefficient* before it, divide all by that Coefficient, and the Equation will be reſolved. Thus if $3x=180$, divide both Sides by 3, and you will have $x=\frac{180}{3}$; that is, $x=60$.

5. If the unknown Quantity, or any other Quantity, be concerned in every Member of the Equation, divide the whole by ſo much as is common, and the Equation

will be reduced to a more *simple* one; thus if $4x^3=196x$, divide the whole by x, and we have $4x^2=196$, whence by the last article $xx=49$.

6. If at any time the unknown Quantity arises to the second Power, or Square of itself, then extract the Square Root on both sides of the Equation; thus $xx=49$; then by extracting the Square Root on both sides, you will have $x=\sqrt{49}$, or rather $x=7$.

EXAMPLES *for clearing simple Equations.*

Suppose this Equation, $\frac{2x}{3}+4=\frac{7x}{12}+9$.

Method of Operation.	Steps	
	1	$\frac{2x}{3}+4=\frac{7x}{12}+9$.
$1\times\overline{3}$	2	$2x+12=\frac{21x}{12}+27$.
$2\times\overline{12}$	3	$24x+144=21x+324$.
$3-21x$	4	$24x-21x+144=324$.
$4-144$	5	$24x-21x=324-144$.
that is	6	$3x=180$.
$6\div\overline{3}$	7	$x=\frac{180}{3}=60$.

First here is given this Equation $\frac{2x}{3}+4=\frac{7x}{12}+9$, to find the Value of x. I set down the Number of the Steps (1) in the Margin, and multiplying every Part of the Equation by 3, it produces a new Step (*viz.* 2) and is thus $2x+12=\frac{21x}{12}+27$, and the marginal Note is $1\times\overline{3}$, denoting the first Step to be multiplied quite through by the Denominator 3. Then I observe my Equation in the 2d Step; and find another Fraction (*viz.* $\frac{21x}{12}$), by whose Denominator I multiply the whole Equation in the 2d Step, and it produces $24x+144=21x+324$, and in the Margin I set $\overline{2}\times\overline{12}$, which makes a third Step.

Now

Now having got my Equation out of Fractions, I must so transpose the Terms by *Addition*, *Subtraction*, &c. till I bring the unknown Quantity on one side of the Equation with an affirmative Value as the 5th Step, which order'd according to the Signs $+$ and $-$, gives $3x=180$ for the 6th Step; then by *Article* 4. above, x will be $=60$; and so of any other.

	Steps	
Equation.	1	$\frac{x}{6}-4=24-\frac{x}{8}$. What is $x=$?
$1\times\overline{6}$	2	$x-24=144-\frac{6x}{8}$.
$2\times\overline{8}$	3	$8x-192=1152-6x$.
$3+6x$	4	$8x+6x-192=1152$.
$4+192$	5	$14x=1152+192=1344$.
$5\div\overline{14}$	6	$x=\frac{1344}{14}=96$.

Given this Equation $a-\frac{x}{b}=\frac{9x}{c}-b$; to find the Value of x.

Thus	1	$a-\frac{x}{b}=\frac{9x}{c}-b$.
$1\times b$	2	$ab-x=\frac{9bx}{c}-bb$.
$2\times c$	3	$abc-cx=9bx-b^2c$.
$3+cx$	4	$abc=9bx+cx-b^2c$
$4+b^2c$	5	$9bx+cx=abc+b^2c$.
$5\div 9b+c$	6	$x=\frac{abc+b^2c}{9b+c}$.

Given this Equation $56-\frac{3x}{4}=48-\frac{5x}{8}$; to find the Value of x, proceed thus;

	Steps	
	1	$56-\frac{3x}{4}=48-\frac{5x}{8}$.
$1\times\overline{4}$	2	$224-3x=192-\frac{20x}{8}$.
$2\times\overline{8}$	3	$1792-24x=1536-20x$.
$3+24x$	4	$1792=1536-20x+24x$.
that is	5	$4x+1536=1792$.
$5-1536$	6	$4x=1792-1536=256$.
$6\div\overline{4}$	7	$x=\frac{256}{4}=64$.

Given this Equation $\frac{7x-5}{8}=\frac{9x}{10}-8$; to find the Value of x. Thus

	1	$\frac{7x}{8}-5=\frac{9x}{10}-8$.
$1\times\overline{8}$	2	$7x-40=\frac{72x}{10}-64$.
$2\times\overline{10}$	3	$70x-400=72x-640$.
$3+640$	4	$640+70x-400=72x$.
$4-70x$	5	$640-400=72x-70x$.
that is	6	$240=2x$.
$6\div\overline{2}$	7	$x=\frac{240}{2}=120$.

Given this Equation $\frac{ax}{bc}-a+b=c-\frac{dx+ac}{abc}$; to find the Value of x, when $a=12$, $b=1$, $c=6$, $d=4$.

Thus	1	$\frac{ax}{bc}-a+b=c-\frac{dx+ac}{abc}$.
$1\times bc$	2	$ax-abc+bbc=bcc-\frac{bcdx+abcc}{abc}$.
$2\times abc$	3	$aabcx-a^2b^2c^2+ab^3c^2=ab^2c^3-bcdx+abcc$.
$3+bcdx$	4	$a^2bcx+bcdx-a^2b^2c^2+ab^3c^2=ab^2c^3+abc^2$.
$4+a^2b^2c^2$	5	$a^2bcx+bcdx+ab^3c^2=ab^2c^3+abc^2+a^2b^2c^2$.
$5-ab^3c^2$	6	$a^2bcx+bcdx=ab^2c^3+abc^2+a^2b^2c^2-ab^3c^2$.
$6\div$	7	$x=\frac{ab^2c^3+abc^2+a^2b^2c^2-ab^3c^2}{a^2bc+bcd}=\frac{279936}{7104}=$ 39.4.

General Directions for solving Mathematical Problems.

WHEN a Question is proposed to be resolved algebraically, put Letters for all the Quantities concern'd therein, whether known or unknown, according to the algebraic Method of Notation, and suppose that the Thing required is done. Then proceed exactly as if the Answer was really known, and you were going to try whether it is true or not; and this is to be done by truly stating all the Conditions of the Question, which will furnish you with one or more Equations, that must be cleared by the Rules given before in the *Reduction of Equations*.

For if the Signs in Algebra be rightly understood, the very writing down the Conditions of the Problem will furnish you with a proper Number of Equations, by which how to know whether the Problem be limited or not. Observe the following Rules.

RULE I.

When the Number of Quantities sought exceeds the Number of given Equations, not depending on one another, the Question is capable of innumerable Answers.

RULE

RULE II.

When the Number of the Equations, not depending on one another, are just as many as the Number of the Quantities sought, then is the Equation truly limited.

SCHOLIUM I.

Problems are more than limited, when the Number of unknown Quantities is less than the Number of independent Equations, in which Case it often becomes impossible.

SCHOLIUM II.

A Problem will be impossible, tho' the Number of Equations be less than the Number of unknown Quantities, if they involve any contradiction; as if a, e, y, be three unknown Quantities, and if $a - e + 2y = b$.
and $2a - 2e + 4y = 3b$.

Now since, by the first Equation, $2a - 2e + 4y = 2b$, therefore $2b$ would be $= 3b$, which is a Contradiction, if b be any real Quantity.

The Solution *of* Equations.

Of Simple *or* Pure Equations.

THE Reader having now well acquainted himſelf with the preceding Rules, it may not be improper, if we come now to ſhew the Manner of applying the ſame in ſolving mathematical Problems; to do which we have thought it neceſſary here, to inſert ſome of the eaſieſt Problems we could think of, and if the Method of Solution, we have uſed, be well obſerved, our young Learners will find no Difficulty in reſolving thoſe that are more intricate.

PROBLEM I.

A Man being 100 *Years old, upon his Birth-Day had his three Sons with him at Dinner, namely,* William, James *and* Thomas. *The Father ſaying to them, Well, Sons, I am this Day juſt* 100 *Years old*; William *the youngeſt, ſaid, Father, my Brother* Thomas *is four times as old as I am, and my Brother* James *three times as old as I am, and all our Ages together are juſt your Age. How old was each of the three Sons?*

SOLUTION.

Put $x =$ *William*'s Age; then becauſe *Thomas* is four times older than *William*, his Age will be $4x$; and becauſe *James* is three times as old as *William*, his Age will be denoted by $3x$; now *William* ſays that all their Ages added together is juſt the Father's Age, or 100.

For	1	$x =$ *William*'s Age.
Then	2	$4x =$ *Thomas*'s Age.
And	3	$3x =$ *James*'s Age.
$1+2+3$	4	$8x = 100$ by the Queſtion.
$4 \div 8$	5	$x = \frac{100}{8} = 12$ Years 6 Months *William*'s Age.

Conſequently $4x = 50$ Years, *Thomas*'s Age, and $3x =$ 37 Years 6 Months, *James*'s Age.

PROB-

PROBLEM II.

THERE *were in Company together four Persons,* Adam, Edward, Charles *and* William. Adam *told* Edward *that he was older than him by two Years*; Charles *told them that he was as old as both of them together, and four Years over.* William *hearing them say so, said, I am just* 96 *Years old, and that is equal to all your Ages. How old were* Adam, Edward, *and* Charles?

SOLUTION.

Let *Edward*'s Age be denoted by x; then *Adam* being 2 Years older, his Age will be $x+2$; and *Charles* being as old as both, and 4 Years over, his Age will be $x+x+2+4$, that is, $2x+6$. But *William* said all their ages added together was just equal to his.

$$\text{For}+\left\{\begin{array}{c|l} 1 & x = \textit{Edward}\text{'s} \\ 2 & x+2 = \textit{Adam}\text{'s} \\ 3 & 2x+6 = \textit{Charles}\text{'s} \end{array}\right\}\text{Age} = \left\{\textit{William}\text{'s}\right.$$

$$\begin{array}{r|c|l} 1+2+3 & 4 & 4x+8=96 \text{ Equation.} \\ 4-8 & 5 & 4x=96-8=88 \\ 5\div 4 & 6 & x=\frac{88}{4}=22, \textit{ Edward}\text{'s Age} \\ \text{Consequently} & 7 & x+2=24, \textit{ Adam}\text{'s Age} \\ \text{And} & 8 & 2x+6=50, \textit{ Charles}\text{'s Age.} \end{array}$$

PROBLEM III.

THREE *Persons,* Andrew, Benjamin *and* Charles, *are to go a Journey of* 340 *Miles*; *of this Journey,* Andrew *is to go a certain Number of Miles unknown*; Benjamin *is to go four times as many Miles as* Andrew, *and three Miles more*; *and* Charles *is to go twice as many Miles as* Benjamin, *and five Miles more. How many Miles must each of these Persons travel severally, to make out their Journey* 340 *Miles?*

SOLUTION.

Put $x=$ Number of Miles that *Andrew* went; then *Benjamin* will go $4x+3$, and *Charles* will go $8x+6+5$, or

or $8x+11$, because he goes twice as many miles, and five more than *Charles*; all which added together must be equal to 340, whence this

Equation	1	$13x+14=340$
$1-14$	2	$13x=340-14=326$
$2\div13$	3	$x=\frac{326}{13}=25\frac{1}{13}$, the Miles *Andrew* went
Then	4	$103\frac{4}{13}$ the Miles that *Benjamin* went, and

$211\frac{8}{13}$ the Miles that *Charles* went.

$$\text{For}\begin{cases}25\frac{1}{13}\\103\frac{4}{13}\\211\frac{8}{13}\end{cases}$$

Proof 340

PROBLEM IV.

THERE is 273l. *to be divided amongst* 4 *Persons, namely*, Andrew, Bennet, Christopher *and* Daniel. Andrew *is to have a Share unknown*; Bennet *is to have twice as much as* Andrew, *and* 30l. *more*; Christopher *is to have* 3 *times as much as* Andrew, *wanting* 52l. *and* Daniel *is to have five times as much as* Andrew, *and* 20l. *more. How must this* 273l. *be divided amongst them, that every one may have his true Share?*

SOLUTION.

Let x denote *Andrew*'s Share; then *Bennet*'s will be $2x+30$, *Christopher*'s will be $3x-52$, and *Daniel*'s $5x+20$, according to the Nature of the Question; which being added together, the Sum must be equal to 273*l.*, whence this

Equation	1	$11x+30+20-52=273$
that is	2	$11x-2=273$
$2+2$	3	$11x=273+2=275$
$3\div11$	4	$x=\frac{275}{11}=25$, *Andrew*'s Share

Whence *Bennet*'s $=80$, *Christopher*'s $=23$, and *Daniel*'s $=145$.

$$\text{For}\begin{cases}25\\80\\23\\145\end{cases}$$

273

PROB-

PROBLEM V.

A Man dying, gave to his eldest Son $\frac{2}{3}$ of $\frac{1}{4}$ of his Estate, to his second Son $\frac{1}{5}$ of $\frac{1}{2}$ of his Estate; and when they had counted their Portions, the eldest Son had 40l. *more than the other: the remainder of the Estate was given to the Wife and younger Children. The Question is, how much was the first and second Son's Portion? and how much was left to the Wife and younger Children?*

SOLUTION.

Let x be put for the whole Estate; then $\frac{2}{3}$ of $\frac{1}{4}$ of x is $\frac{2}{12}x$, or $\frac{1}{6}x=$ the eldest Son's Share; and $\frac{1}{5}$ of $\frac{1}{2}$ of x, or $\frac{1}{10}x$, is the second Son's Share; but the Question says, the eldest Son had 40*l.* more than the second; therefore to bring it to an Equation, add 40*l.* to the second Son's Share, and it will be $\frac{1}{10}x+40$; which, by the Question, is equal to the eldest Son's Share; whence this

Equation	1	$\frac{1}{6}x=\frac{1}{10}x+40$
$1-\frac{1}{10}x$	2	$\frac{1}{6}x-\frac{1}{10}x=40$
that is	3	$\frac{1}{15}x=40$
3×15	4	$x=600l.$ the whole Estate.

Then $\frac{1}{6}x=100l.$ the eldest Son's Share.
$\frac{1}{10}x=60l.$ the second Son's Share.
And there was 440*l.* for the Wife and younger Children.

PROBLEM VI.

ONE coming into an Orchard asked the Gardiner how many Trees there were in the Orchard; the Gardiner answered, that the one half of the Trees were Apple-Trees, a fourth Part Pear-Trees, the seventh Part were Plumb-Trees, and that there were 12 *Cherry-Trees besides. How many Trees were there in the Orchard?*

SOLUTION.

Put $x=$ the Number of Trees that were in the Orchard; then $\frac{x}{2}$ will be the Number of Apple-Trees,

x

$\frac{x}{4}$ the Pear-Trees, $\frac{x}{7}$ the Plumb-Trees, and 12 Cherry-Trees; all which being added together must, by the Question, be equal to x, the total Number of Trees in the Orchard; which being done, we have this

Equation	1	$\frac{x}{2}+\frac{x}{4}+\frac{x}{7}+12=x$
Contrac.	2	$\frac{50x}{56}+12=x$, by the Rules of *Fractions*.
$3\times\overline{56}$	3	$50x+672=56x$
$4-50x$	4	$56x-50x=672$
that is	5	$6x=672$
$6\div\overline{6}$	6	$x=\frac{672}{6}=112$, the Number of Trees in the Orchard.

Whence there were 56 Apple-Trees, 28 Pear-Trees, 16 Plumb-Trees, which added to the 12 Cherry-Trees make 112.

PROBLEM VII.

A. begins the World with a certain Stock of Money, which he improv'd so well by Trade, that at the End of the first Year he doubled his Stock, except 100l. *expended for the Use of his Family; and so he goes on every Year doubling his last Year's Stock, except* 100l. *a Year's Expences, and at the End of three Years he found himself three times as rich as at first. I demand his first Stock?*

SOLUTION.

Put $x=$ his first Stock; then his Stock at the End of the first Year is $2x-100$

Second Year's End	$4x-300$
Third Year's End	$8x-700$
Equation	$8x-700=3x$

Equation	1	$8x-700=3x$
$1+700$	2	$8x=3x+700$
$2-3x$	3	$8x-3x=700$
that is	4	$5x=700$
$4\div\overline{5}$	5	$x=\frac{700}{5}=140$

Proof. For $140\times 8=1120$, and $1120-700=420$, and $140\times 3=420$.

PROBLEM VIII.

THERE is a Fish whose Head is 9 *Inches long, and the Tail as long as the Head and half the Body, and the Body as long as the Head and Tail together.* Quere, *the length of the Body and Tail separately?*

SOLUTION.

The length of the Body put x.

Then the length of the Tail will be $\frac{x}{2}+9$.

Equation $x=\frac{x}{2}+9+9$.

Equation	1	$x=\frac{x}{2}+18$
1×2	2	$2x=x+36$
$2-x$	3	$x=36$ the length of the Body.

and 27 the length of the Tail.

PROBLEM IX.

REquired to divide 100 *into two such Parts, that* $\frac{3}{4}$ *of one Part subtracted from* $\frac{5}{6}$ *of the other may leave* 39?

SOLUTION.

For the first Part put	x
Then by Ax. 3. the other is	$100-x$
$\frac{3}{4}$ of one Part	$\frac{3x}{4}$
And $\frac{5}{6}$ of the other	$\frac{500-5x}{6}$

Equation $\frac{500-5x}{6}-\frac{3x}{4}=39$.

Equation

Equation	1	$\frac{500-5x}{6}-\frac{3x}{4}=39$
$1\times\overline{6}$	2	$500-5x-\frac{18x}{4}=234$
$2\times\overline{4}$	3	$2000-20x-18x=936$
$3+20x$	4	$2000-18x=936+20x$
$4+18x$	5	$2000=936+20x+18x$
$5-936$	6	$2000-936=20x+18x$
that is	7	$38x=1064$
$7\div\overline{38}$	8	$x=\frac{1064}{38}=28.$

PROBLEM X.

A *and* B *engage at Play*, A *had* 100 *Guineas*, *and* B 80, *before they begin*; *and*, *after a certain Number of Games won and lost between them*, A *rises with three times as many Guineas as* B; *I demand how many Guineas* A *won of* B?

SOLUTION.

The Number *A.* won of *B.* put	$x.$
A.'s Number at last	$x+100.$
B.'s at last	$80-x.$

Equation $100+x=240-3x.$

Equation	1	$100+x=240-3x$
$1+3x$	2	$100+4x=240.$
$2-100$	3	$4x=140.$
$3\div\overline{4}$	4	$x=\frac{140}{4}=35.$

PROBLEM XI.

ONE meeting a Company of Beggars, gave to each 6 *Pence, and had* 20 *Pence over*; *but if he had given* 8 *Pence each, he would have wanted* 16 *Pence for that Purpose.* Quere, *the Number of Beggars?*

SOLUTION.

The Number of Beggars put	$x.$
The Number of Pence given	$6x.$
The Number of Pence in all	$6x+20.$

The Number of Pence that would have been given at 8 Pence } $8x$

Then he would have wanted 16 Pence, *viz.* $8x-16$

Equation $6x+20=8x-16$.

Equation	1	$6x+20=8x-16$.
$1+16$	2	$6x+36=8x$.
$2-6x$	3	$8x-6x=36$.
that is	4	$2x=36$.
$4\div 2$	5	$x=\frac{36}{2}=18$.

For $18\times6=108$.
$+\ 20$
128

And $18\times8=144$
$-\ 16$
128

PROBLEM XII.

ONE *lets out a certain Sum of Money at* 4 per Cent. *Simple Intereſt, which, in* 12 *Years, wanted but* 20l. *of the Principal.* What was the Principal?

SOLUTION.

For the Principal put x

The Intereſt for a Year, For $100 : 4 :: x : \frac{4x}{100}$ } $\frac{4x}{100}$

For 12 Years $\frac{48x}{100}$ or $\frac{12}{25}x$.

Equation $\frac{12}{25}x=x-20$.

Equation	1	$\frac{12}{25}x=x-20$.
1×25	2	$12x=25x-500$.
$2+500$	3	$500+12x=25x$.
$3=12x$	4	$500=25x-12x$.
hence	5	$x=\frac{500}{13}=38\frac{6}{13}$.

PROBLEM XIII.

REquired to divide 24s. *into* 24 *Pieces, conſiſting only of Nine-pences, and Thirteen Pence Half-pennies?*

SOLU-

SOLUTION.

For the Number of Nine-pences put	x
By *Ax.* 3*d.* of 13*d.* $\frac{1}{2}$	$24-x$
Of Half-pence = the former	$18x$
The latter	$648-27x$
The whole	$18x+648-27x$

Equation $18x+648-27x=576$.

Equation	1	$18x+648-27x=576$.
$1+27x$	2	$18x+648=576+27x$.
$2-18x$	3	$648=576+9x$.
$3-576$	4	$72=9x$.
$4\div 9$	5	$x=\frac{72}{9}=8$ Nine-pences, and $24-8=16$, 13*d* $\frac{1}{2}$

To ſtrengthen the Idea of the Learner, we have all along ſuppoſed x for the unknown Quantity, and the known ones by their reſpective Figures. But the ſame may be done more generally, and at the ſame time more elegantly, by ſubſtituting a, b, c, d, &c. for known Quantities, and x, y, z, v, &c. for unknown. Beſides, it will be more conducive to *reaſoning*, becauſe it ſerves *univerſally*, in all Caſes, and does not in the leaſt appear as if *contrived*, and therefore is preferable to any other, to repreſent the known and unknown Quantities by Algebraic Symbols; ſince from thence a general Theorem is derived, whereby all other Queſtions of the ſame kind may be reſolved.

PROBLEM XIV.

TWO Men, A. *and* B. *travelling together,* A. *with* 240l. *and* B. *with* 96l. *met a Company of Robbers, who took twice as much from* A. *as from* B. *and left* A. *thrice as much as they left* B. *How much did they take from each?*

SOLUTION.

Put	$240=a$
	$96=b$
	$2=c$
	$3=d$
Sum taken from B	$=x$

Then

Then from A $=cx$

By *Ax*. 3 $\begin{cases}\text{Left B} & =b-x\\ \text{Left A} & =a-cx\end{cases}$

Equation $a-cx=db-dx$.

Equation	1	$a-cx=db-dx$
$1+dx$	2	$a-cx+dx=db$.
$2-a$	3	$dx-cx=db-a$.
$3\div$	4	$x=\dfrac{db-a}{d-c}=48$.

PROBLEM XV.

SUPPOSE a Cask holds 81 Gallons of Wine when full, out of which a certain Quantity is exhausted, and then the Cask is filled up again with Water; the same Quantity being again drawn out as at first, and the Cask again filled up with Water, and so on four Times, always filling the Cask with Water after every Evacuation, there is at last found 16 Gallons of Wine left in the Cask besides Water. What Quantity of Wine was drawn out each Time?

SOLUTION.

Since there was the same Quantity of Liquor in the Vessel before every Draught was made, *viz.* 81 Gallons, and since there was taken the same Quantity at every Draught, it follows that there must be left the same Quantity after every Draught. Call this remainder x, and then we may observe, that at every Draught the whole Quantity of Liquor in the Vessel, and consequently the Wine, will be diminish'd in the Proportion of 81 to x, and tho' the Water was afterwards recruited, the Wine was not; therefore if x represents the Wine left in the Vessel after the first Draught, and a be put $=81$, $b=16$, the Wine after the second Draught may be found by saying

As $a : x :: x : \dfrac{x^2}{a} =$ the Wine left after the second Draught.

Also $a : x :: \dfrac{x^2}{a} : \dfrac{x^3}{a^2} =$ Wine left after the 3d Draught.

And

And $a : x :: \frac{x^3}{a^2} : \frac{x^4}{a^3} =$ Wife left after the fourth Draught, which by the Question must be equal to 16 Gallons. Hence this

Equation $\frac{x^4}{a^3} = b$.

Equation	1	$\frac{x^4}{a^3} = b$.
$1 \times a^3$	2	$x^4 = a^3 b$
2 ω 2	3	$x^2 = \sqrt{a^3 b}$
3 ω 2	4	$x = \overline{\sqrt{a^3 b}}\vert^{\frac{1}{2}} = 54$.

For $\frac{x^4}{531441} = 16$

$$x^4 = 8503056$$
$$x^3 = 157464$$
$$x^2 = 2916$$
$$x = 54$$

PROOF.

81—54=27, first Draught.
54—36=18, second Draught.
36—24=12, third Draught.
24—16= 8, fourth Draught.
65+16=81. *Q. E. I.*

PROBLEM XVI.

A Clock has two Hands turning upon the same Center, whereof the swifter makes a Revolution every 12 *Hours, and the slower every* 16. Quere, *the Synodical Period of the two Hands?*

N. B. That by a synodical Period, I mean the whole Time from the Moment the two Hands are together to the Moment they come together again; whence it follows that, in every synodical Period, the swifter Hand makes one Revolution more than the slower. For if we once suppose the two Hands together, it will be impossible for them to come together again, till the swifter Hand has got an entire Revolution before the slower, in which time it must have made one Revolution more than the slower; whence the Solution is as follows:

SOLU-

SOLUTION.

Put $12=a$, $16=b$, difference $=d=4$.

Number of Hours in a synodical Period $=x$.

Number of Revolutions in the swifter $=\frac{x}{a}$.

In the Slower $=\frac{x}{b}$.

The Difference $=\frac{x}{d}$.

Equation $\frac{x}{a}=\frac{x}{b}+1$.

Equation	1	$\frac{x}{a}=\frac{x}{b}+1$.
$1 \times a$	2	$x=\frac{ax}{b}+a$.
$2 \times b$	3	$bx-ax+ab$.
$3-ax$	4	$bx-ax=ab$.
$4 \div b-a$	5	$x=\frac{ab}{b-a}=48$.

PROBLEM XVII.

ONE buys a certain Number of Eggs, half at 2 a Penny, and the other half at 3 a Penny. Afterwards selling out the Whole at 5 for Two-pence, he contrary to his Expectation lost 12d. *by his Bargain. How many were bought in at first?*

SOLUTION.

Put $12=a$, $2=b$, $3=c$, $5=m$.

Number bought at first x

Number at 2 a Penny $\frac{x}{b}$.

Number at 3 a Penny $\frac{x}{c}$.

Price of the former in Pence $\frac{x}{4}$.

Of the latter $\frac{x}{6}$.

Of

Of the Whole $\frac{10x}{ab}$.

Equation $\frac{10x}{ab}=\frac{bx}{m}+a$.

Equation	1	$\frac{10x}{ab}=\frac{bx}{m}+a$.
$1\times ab$	2	$10x=\frac{abbx}{m}+aab$.
$2\times m$	3	$10mx=abbx+aabm$.
$3-ab^2x$	4	$10mx-abbx=aabm$.
$4\div 10m-abb$	5	$x=\frac{aabm}{10m-abb}=\frac{1440}{2}=720$.

PROBLEM XVIII.

FOUR Gamesters A, B, C, D, *each with a different Stock of Money about him, sit down to play*; A. *wins* $\frac{1}{2}$ *of* B.*'s first Stock, and at the same Time* B. *wins* $\frac{1}{3}$ *of* C.*'s, and* C. $\frac{1}{4}$ *of* D.*'s, and* D. $\frac{1}{5}$ *of* A.*'s; after which they each rise with 23 Guineas. How many Guineas had each at first?*

SOLUTION.

The Money *D.* won of *A.* put	x
A.'s first Stock	$5x$
The Remainder of *A.*'s first Stock after his loss to *D.*	$4x$
The Money *A.* won of *B.*	$23-4x$
B.'s first Stock	$46-8x$
The Remainder of *B.*'s first Stock after his Loss to *A.*	$23-4x$
The Money *B.* won of *C.*	$=4x$
C.'s first Stock	$12x$
Remainder of *C.*'s first Stock after his Loss to *B.*	$8x$
The Money *C.* won of *D.*	$23-8x$
D.'s first Stock	$92-32x$
The Remainder of *D.*'s first Stock after his Loss to *C.*	$69-24x$
Another Expression. *D.* won of *A.*	$=24x-46$

Equation $24x-46=x$.

Equation

Equation	1	$24x-46=x$.
$1+46$	2	$24x=x+46$.
$2-x$	3	$23x=46$.
$3\div 23$	4	$x=\frac{46}{23}=2$, the Money *D.* won of *A.*

PROBLEM XIX.

REquired to divide 180 *into two such Parts that one may be to the other as* 2 *to* 3?

SOLUTION.

Put $2=a$, $3=b$, $180=m$. Let $x=$ one Part, and by Ax. 3. $m-x$ will be the other Part. Proportion $x : m-x :: a : b$. Therefore $bx=am-ax$. By *Prop.* IX.

Equation	1	$bx=am-ax$.
$1-ax$	2	$bx+ax=am$.
$2\div b+a$	3	$x=\dfrac{am}{b+a}=72$.

PROBLEM XX.

THERE is a Pole 6 *Feet long, at whose Extremities are suspended* 2 *Weights, one of* 5 *Pounds, the other of* 7; *I demand the common central Gravity of the Weights, that is, its Distance from the two Extremities of the Pole?*

SOLUTION.

Let 6 *feet* $=72$ *inches* $=a$.

$5=b$, $7=c$.

Dist. of its central Gravity from 7 Pound weight x

From the 5 by *Ax.* 3. $a-x$

Proportion $x : a-x :: b : c$

Therefore $cx=ab-bx$. By *Prop.* IX.

Equation	1	$cx=ab-bx$.
$1+bx$	2	$cx+bx=ab$.
$2\div c+b$	3	$x=\dfrac{ab}{c+b}=30$.

PROB-

PROBLEM XXI.

WHAT Number is that, which being severally added to 36 and 52, will leave the former Sum to the latter as 3 *to* 4?

SOLUTION.

Put $36=a$, $52=b$, $3=c$, $4=d$.
Number sought $=x$.
Proportion $a+x : b+x :: c : d$.
Th. $ad+dx=bc+cx$. By *Prop.* IX.

Equation	1	$ad+dx=bc+cx$.
$1-cx$	2	$ad+dx-cx=bc$.
$2-ad$	3	$dx-cx=bc-ad$.
$3\div d-c$	4	$x=\dfrac{bc-ad}{d-c}=12$.

Of

Of Transferring and exterminating the unknown Quantities.

RULE.

FIND by Reduction the Value of one unknown Quantity in one Equation, and substitute this Value for it in the other Equations.

EXAMPLE.

Let $a+x=2b-y$, and $3ax-yx=d$; to exterminate y. By the first Equation I so transpose the Quantities a and x, till I get $y=2b-a-x$; now seeing $2b-a-x$ is the same as y, substitute this Value of y in the second Equation, and it gives $3ax-x\times\overline{2b-a-x}=d$, that is, $3ax-2bx-ax+xx=d$; but because $3ax-ax$ is $=2ax$, it will be $2ax-2bx+xx=d$.

We have purposely omitted other Examples that fall under this Denomination, because this would swell the Book too big.

PROBLEM XXII.

THERE is a certain Fraction, which, if an Unit be added to the Numerator, will be equal to $\frac{1}{3}$; *but if, on the contrary, an Unit be added to the Denominator, the Fraction then will be equal to* $\frac{1}{4}$. Quere, *the Numerator and Denominator?*

SOLUTION.

For the Fraction sought put $\frac{x}{y}$.

$$\text{Then}\left\{\begin{array}{l|l} 1 & \dfrac{x+1}{y}=\dfrac{1}{3} \\ 2 & \dfrac{x}{y+1}=\dfrac{1}{4} \end{array}\right\}\text{by the Question.}$$

$1\times y$	3	$x+1=\frac{y}{3}$.
3×3	4	$3x+3=y$.
$2\times y+1$	5	$x=\frac{y+1}{4}$.
5×4	6	$4x=y+1$.
$4-3$	7	$3x=y-3$.
$7\div 3$	8	$x=\frac{y-3}{3}$.
$6\div 4$	9	$x=\frac{y+1}{4}$.
$8=9$	10	$\frac{y-3}{3}=\frac{y+1}{4}$, here x is exterminated.
10×3	11	$y-3=\frac{3y+3}{4}$.
11×4	12	$4y-12=3y+3$.
$12+12$	13	$4y=3y+3+12=3y+15$.
$13-3y$	14	$4y-3y=15$.
that is	15	$y=15$.

From the 9th Step $x=4$; so $\frac{4}{15}$ is the Fraction.

PROBLEM XXIII.

THERE is a certain Bowling-green, which, if it was 2 *foot broader, and* 3 *longer, would be* 64 *square Feet; but if, on the other hand, it was* 3 *foot broader, and* 2 *longer, it would be* 68 *square Feet.* Quere *the Dimensions of the Bowling-green?*

SOLUTION.

Let $64=a$, $68=b$, $3=c$, $2=d$, and let x and y denote the Breadth and Length respectively; then its area will be xy, the Breadth and Length of it on the first Supposition will be $x+d$, and $y+c$; now the Length multiplied by the Breadth gives the Area, that is, $\overline{x+d}\times\overline{y+c}=xy+dy+cx+cd$. The Breadth and Length on the second Supposition will be $x+c$ and $y+d$, and for the same Reasons above its Area will be $xy+cy+dx+cd$. Whence we have these

Equations	1	$xy+dy+cx+cd=xy+a.$
	2	$xy+cy+dx+cd=xy+b.$
$1-xy$	3	$dy+cx+cd=a.$
$3-dy$	4	$cx+cd=a-dy.$
$4-cd$	5	$cx=a-dy-cd.$
$5\div c$	6	$x=\frac{a-cd-dy}{c}.$
$2-xy$	7	$cy+dx+cd=b.$
$7-cy$	8	$dx+cd=b-cy.$
$8-cd$	9	$dx=b-cy-cd.$
$9\div d$	10	$x=\frac{b-cd-cy}{d}.$
$6=10$	11	$\frac{a-cd-dy}{c}=\frac{b-cd-cy}{d}$, here x vanishes.
$11\times c$	12	$a-cd-dy=\frac{bc-ccd-ccy}{d}.$
$12\times d$	13	$ad-cdd-ddy=bc-ccd-ccy.$
$13\pm$	14	$ccy-ddy=bc-c^2d+cd^2-ad.$
$14\div$	15	$y=\frac{bc-c^2d+cd^2-ad}{c^2-d^2}=\frac{70}{5}=14.$

Consequently $x=10.$

Note, $\pm$ denotes transposition, and signifies that the 13 step is so order'd as to get the unknown Quantities on one side of the Equation, and the known ones on the other; and is no more but saving the Trouble of setting down a Step or two.

PROBLEM XXIV.

ONE *lays out* 2s. 6d. *in Apples and Pears, buying his Apples at* 4 *a Penny, and his Pears at* 5; *and afterwards accommodates his Neighbour with* $\frac{1}{2}$ *of his Apples, and* $\frac{1}{3}$ *of his Pears for* 13d. *the Price he bought them at.* Quere *how many were bought of each Sort?*

SOLUTION.

Let $2s.$ $6d.=30d.=a$, $4=b$, $5=c$, $13=d$, Number of Apples $=x$, Number of Pears $=y$.

Price

Price of the former in Pence (by *Ax.* 4.) $\frac{x}{b}$

Price of the latter in Pence $\frac{y}{c}$

Price of half the former $\frac{x}{2b}$

Price of one third of the latter $\frac{y}{3c}$

Equations $\begin{cases} \frac{x}{b}+\frac{y}{c}=a. \text{ By } \textit{Reduction}, & x=\frac{acb-by}{c} \\ \frac{x}{2b}+\frac{y}{3c}=d, \text{ And} & x=\frac{6bcd-2by}{3c} \end{cases}$

Whence x is expunged, and we have this

Equation	1	$\frac{acb-by}{c}=\frac{6bcd-2by}{3c}.$
$1\times c$	2	$acb-by=\frac{6bc^2d-2bcy}{3c}.$
$2\times 3c$	3	$3ac^2b-3bcy=6bc^2d-2bcy.$
$3\pm$	4	$bcy=3ac^2b-6bc^2d.$
$4\div bc$	5	$y=\frac{3ac^2b-6bc^2d}{bc}=\frac{1200}{20}=60.$

Hence he bought 60 Pears, and 72 Apples.

PROBLEM XXV.

THERE is a Number, consisting of two Places, which is equal to 4 *Times the Sum of its Digits; and if to the Number be added* 18, *the Digits will be inverted.* Quere *the Number?*

SOLUTION.

The Digit in the Place of Tens	$x.$	$4=a.$
In the Place of Units	$y.$	$18=b.$
Sum of the Digits	$x+y.$	
The Number represented by the Digits	$10x+y$	
The Number represented by the Digits inverted	$10y+x.$	

Equations	1	$10x+y=ax+ay.$
	2	$10y+x=10x+y+b.$
$1-y$	3	$10x=ax+ay-y.$
$3-ax$	4	$10x-ax=ay-y.$
that is	5	$6x=3y.$
$5 \div \bar{6}$	6	$x=\frac{3y}{6}=\frac{y}{2}.$
$2 \pm$	7	$x=\frac{9y-18}{9}.$
$6=7$	8	$\frac{y}{2}=\frac{9y-18}{9}=y-2$, hence x vanishes.
$8 \times \bar{2}$	9	$y=\frac{18y-36}{9}=2y-4.$
$9+\bar{4}$	10	$y+4=2y.$
$10-y$	11	$y=4.$

Hence 24 the Number sought
42 the Digits inverted

LEMMA.

In all Problems of three unknown Quantities, we must find three several Values of the first; then comparing these together, and resolving them, find two Values of the second; lastly, by comparing these two Values, find the value of the third.

So if there were four unknown Quantities, we must have four Equations of the first, three of the second, two of the third, and one of the last, or fourth, all found in the same Manner as above directed.

PROBLEM XXVI.

THREE Persons, A. B. *and* C. *were talking of their Money; says* A. *to* B. *and* C. *give me* $\frac{1}{2}$ *of your Money, and I shall have* 17 *Guineas; says* B. *to* A. *and* C. *give me* $\frac{1}{3}$ *of your Money, and I shall have* 17 *Guineas; says* C. *to* A. *and* B. *give me* $\frac{1}{4}$ *of your Money, and I shall have* 17 *Guineas.* Quere *the Number of Guineas each Man had?*

SOLU-

SOLUTION.

For the Number of *A*'s Guineas put x.
Of *B*'s y.
Of *C*'s z.

$$\text{Equations}\begin{cases} x+\dfrac{y+z}{2}=17. \\ y+\dfrac{x+z}{3}=17. \\ z+\dfrac{x+y}{4}=17. \end{cases}$$

1st Equation	1	$x+\dfrac{y+z}{2}=17.$
Reduced	2	$x=\dfrac{34-y-z}{2}.$

2d Equation	1	$y+\dfrac{x+z}{3}=17.$
Reduced	2	$x=51-3y-z.$

3d Equation	1	$z+\dfrac{x+y}{4}=17.$
Reduced	2	$x=68-y-4z.$

Now from the above three Equations we have,

First, $\dfrac{34-y-z}{2}=51-3y-z$, here x vanishes.

Reduced, $y=\dfrac{68-z}{5}$.

Secondly, $51-3y-z=68-y-4z$.

Reduced, $y=\dfrac{3z-17}{2}$.

Lastly, $\dfrac{68-z}{5}=\dfrac{3z-17}{2}$; here y vanishes.

Reduced, $z=\frac{221}{17}=13$ Guineas, *C*'s Number.
Whence $y=11$ Guineas, *B*'s Number.
And $x=5$ Guineas, *A*'s Number.

Of the Resolution of adfected Quadratic Equations.

AN adfected Quadratic Equation is an Equation including three different Sorts of Quantities; one Sort wherein the Square of the unknown Quantity is concerned; another Sort wherein it is simply concerned; and a third Sort wherein it is not concerned at all; as if $24x - 2xx = xx + 45$. For a Resolution whereof, we are first to say something of a *Binomial*: Now a Binomial is a Quantity consisting of two Members, connected together by the signs $+$ or $-$; as $x+a$, or $a-x$, or $x+\frac{a}{2}$, or $x-\frac{a}{2}$, &c. And a Square raised from a Binomial Root, is nothing else but the Square of such a Quantity.

Thus the Square of $x+\frac{a}{2}$ is $\overline{x+\frac{a}{2}}|^2 = xx+ax+\frac{aa}{4}$, and that of $x-\frac{a}{2}$ is $\overline{x-\frac{a}{2}}|^2 = xx-ax+\frac{aa}{4}$; whence we may observe,

First, That whenever we meet with any Quantity consisting of two Members, as $xx+ax$, or $xx-ax$, whereof one, as xx, is a Square, and the other $+ax$ is the Root of the Square multiplied into some given Coefficient $\pm a$; whenever, I say, we meet with such a Quantity, it may be considered as the two first Members of an imperfect Square, raised from a Binomial Root, and may be easily compleated by adding $\frac{aa}{4}$, that is, *by adding the Square of half the Coefficient of the Second.* Thus, $xx+6x$, when compleated, becomes $xx+6x+9$; and $xx-8x$, when compleated, becomes $xx-8x+16$.

Secondly, it may be observed that the Root of such a Square, thus compleated, will always be the Root of the first

firſt Member, together with half the Coefficient of the Second. Thus the Square-Root of $xx+6x+9$ is $x+3$, and that of $xx-8x+16$ is $x-4$. And this premis'd, the Reſolution of Quadratic Equations is as follows.

By compleating the Square.

RULE.

ADD the Square of half the Coefficient of the unknown Quantity, to each ſide of the Equation, and the Square will be compleat.

EXAMPLES.

		When compleated.		Their Roots.
1.	x^2+10x		$x^2+10x+25$	$x+5$
2.	x^2+12x		$x^2+12x+36$	$x+6$
3.	$x^2+\ 3x$		$x^2+12x+\frac{9}{4}$	$x+\frac{3}{2}$
4.	$x^2+\ 5x$		$x^2+\ 5x+\frac{25}{4}$	$x+\frac{5}{2}$

In reſolving a Quadratic Equation, you muſt firſt of all clear it from Fractions, by the Rule already laid down, and by that means bring it into Integers; having ſo done, tranſpoſe the Terms, ſo that the two Powers of the unknown Quantity may ſolely poſſeſs one Side of the Equation, *viz.* that Side which will exhibit the Square or higheſt Power of the unknown Quantity. This Rule obſerved in the following Equation, we ſhall have $3xx-24x=-45$. In the next Place, ſince the higheſt Power, or xx, has a Coefficient before it, as 3, free it from the Coefficient, by dividing the whole Equation by 3, and we ſhall have $xx-8=-15$; and $xx-8x$ muſt be conſidered as the two firſt Members of an imperfect Square, rais'd from a Binomial Root, which being compleated, according to the Rule above, we ſhall have $xx-8x+16=1$; and, conſequently, the Square Root of one Side being equal to the Square Root of the other, we have $x-4=\pm 1$; I ſay, ± 1, becauſe either $+1$, or -1, multiplied into itſelf, produces $+1$; but you tranſpoſe 4, to make it affirmative, and then $x=4\pm 1$; ſo that in the Equation $x=3$ or 5, each of which will equally ſatisfy the Conditions of the Problem. Whence it may be

be observed, that every Quadratic Equation admits of two Roots, or Numbers, that will answer it; and consequently all Problems, producing Quadratic Equations, do necessarily admit of two Solutions, which in Geometrical Problems, and in many others, are equally significant, tho' in Arithmetical Questions they are not equally useful, I mean when the Root of the Equation is negative.

N. B. *c*, □, *set in the Margin of the Register Steps, denote the Square compleated.*

There are three Cases that fall under the Denomination of Quadratics with regard to finding the true Root or Value of the unknown Quantity, and they are known by the Signs $+$ and $-$.

CASE I.

If $xx+ax=b$. Then $x=\sqrt{b+\frac{1}{4}a^2}-\frac{1}{2}a$,

For	1	$xx+ax=b$.
1 *c* □	2	$xx+ax+\frac{1}{4}a^2=b+\frac{1}{4}a^2$.
2 ꞷ 2	3	$x+\frac{1}{2}a=\sqrt{b+\frac{1}{4}a^2}$
$3-\frac{1}{2}x$	4	$x=\sqrt{b+\frac{1}{4}a^2}-\frac{1}{2}a$.

CASE II.

If $xx-ax=b$. Then $x=\sqrt{b+\frac{1}{4}a^2}+\frac{1}{2}a$.

For	1	$xx-ax=b$.
1 *c* □	2	$xx-ax+\frac{1}{4}a^2=b+\frac{1}{4}a^2$.
2 ꞷ 2	3	$x-\frac{1}{2}a=\sqrt{b+\frac{1}{4}a^2}$.
$3+\frac{1}{2}a$	4	$x=\sqrt{b+\frac{1}{4}a^2}+\frac{1}{2}a$, or $=\frac{1}{2}a+\sqrt{b+\frac{1}{4}a^2}$.

CASE III.

If $ax-xx=b$, or $xx-ax+b=0$, or $xx-ax=-b$. Then $x=\frac{1}{2}a\pm\sqrt{\frac{1}{4}a^2-b}$.

For	1	$xx-ax=-b$.
1 *c* □	2	$x^2-ax+\frac{1}{4}a^2=\frac{1}{4}a^2-b$.
2 ꞷ 2	3	$x-\frac{1}{2}a=\sqrt{\frac{1}{4}a^2-b}$.
$3+\frac{1}{2}a$	4	$x=\frac{1}{2}a\pm\sqrt{\frac{1}{4}a^2-b}$.

Of

Quadratics of an higher Nature.

If $x^4+ax^2=b^2$. Then $x=\sqrt{:\sqrt{b^2+\frac{1}{4}a^2}-\frac{1}{2}a}$.

For	1	$x^4+ax^2=b^2$.
1 c □	2	$x^4+ax^2+\frac{1}{4}a^2=b^2+\frac{1}{4}a^2$.
3 ω 2	3	$x^2+\frac{1}{2}a=\sqrt{b^2+\frac{1}{4}a^2}$.
$3-\frac{1}{2}a$	4	$x^2=\sqrt{b^2+\frac{1}{4}a^2}-\frac{1}{2}a$.
4 ω 2	5	$x=\sqrt{:\sqrt{b^2+\frac{1}{4}a^2}-\frac{1}{2}a}$.

If $x^6+ax^3=b^2$. Then $x=\sqrt[3]{:\sqrt{b^2+\frac{1}{4}a^2}-\frac{1}{2}a}$.

For	1	$x^6+ax^3=b^2$.
1 c □	2	$x^6+ax^3+\frac{1}{4}a^2=b^2+\frac{1}{4}a^2$.
2 ω 2	3	$x^3+\frac{1}{2}a=\sqrt{b^2+\frac{1}{4}a^2}$.
$3-\frac{1}{2}a$	4	$x^3=\sqrt{b^2+\frac{1}{4}a^2}-\frac{1}{2}a$.
4 ω 3	5	$x=\sqrt[3]{:\sqrt{b^2+\frac{1}{4}a^2}-\frac{1}{2}a}$, or $\overline{\sqrt{b^2+\frac{1}{4}a^2}-\frac{1}{2}a}\Big\vert^{\frac{1}{3}}$.

PROBLEM XXVII.

WHAT two Numbers are those, whose Sum is 27, *and the Sum of their Squares* 449?

SOLUTION.

Let $27=a$; $449=b$.

Let one Number be	x
Then *per Ax.* 3. the other is	$a-x$
The Square of the former	xx
Of the latter	$aa-2ax+xx$
Sum of their Squares	$aa-2ax+2xx$

Equation	1	$aa-2ax+2x^2=b$.
$1-a^2$	2	$2x^2-2ax=b-a^2$.
$2\div 2$	3	$x^2-ax=\frac{b-aa}{2}$.
3 c □	4	$x^2-ax+\frac{1}{4}aa=\frac{1}{4}aa+\frac{b-aa}{2}$.
4 ω 2	5	$x-\frac{1}{2}a=\sqrt{\frac{1}{4}aa+\frac{b-aa}{2}}$.
$5+\frac{1}{2}a$	6	$x=\frac{1}{2}a+\sqrt{\frac{1}{4}aa+\frac{b-aa}{2}}=\frac{27\pm13}{2}=20$ or 7.

PROBLEM XXVIII.

A Man buys a Horse, and sells him again for 24 l. *and gain'd as much* per Cent. *as the Horse cost him; what cost the Horse?*

N. B. Gain'd as much *per Cent.* is to be underſtood thus; he gain'd ſo much, that 100*l.* at the ſame rate of Gain would have gain'd the Money he laid out.

SOLUTION.

For the Money laid out put x.
Then by *Ax.* 3. the gain $24-x$.

Proportion $x : 24-x :: 100 : \frac{2400-100x}{x} =$ the Gain *per Cent.*

Equation	1	$\frac{2400-100x}{x}=x$.
$1 \times x$	2	$2400-100x=xx$.
$2+100x$	3	$xx+100x=2400$.
$3 \, c \, \square$	4	$xx+100x+2500=4900$.
$4 \, \omega\!\omega \, 2$	5	$x+50=\sqrt{4900}=70$.
$5-50$	6	$x=70-50=20$, the Horſe coſt him.

He laid out 20*l.* and gain'd 4*l.* and 100*l.* at that Rate of Intereſt would have gain'd 20*l.* which is the Money he laid out.

For $20 : 4 :: 100 : 20$.

PROBLEM XXIX.

WHAT two Numbers are those, whereof twice the first, with three times the second, make 60, *and twice the Square of the first, with* 3 *Times the Square of the second, make* 840?

SOLUTION.

Let x and y denote the two Numbers ſought.

Then $2x+3y=60$; Theref. $x=\frac{60-3y}{2}$.

Alſo

Also $2x^2+3y^2=840$; Theref. $x^2=\frac{840-3yy}{2}$.

Equation	1	$\left(\frac{60-3y}{2}\right)^2=\frac{840-3yy}{2}$.
that is	2	$\frac{3600-360y+9yy}{4}=\frac{840-3yy}{2}$.
$2\times$ &c.	3	$7200-720y+18yy=3360-12yy$.
$3\pm$	4	$30yy-720y=-3840$.
$4\div\overline{30}$	5	$yy-24y=-128$.
5 c □	6	$y^2-24y+144=144-128=16$.
6 ɯ 2	7	$y-12=\sqrt{16}=4$.
$7+12$	8	$y=12\pm4=16$ or 8.
And	9	$x=18$ or 6.

To find any Term in Arithmetical Progression.

IN Arithmetical Progression, any Term required may be made (without making all the antecedent Terms) by adding the first Term to the Product of the Difference (that is, the Number whereby all the Terms exceed one another) multiplied into the Distance of the Term required from the first. For Instance, in this Arithmetical Progression, 2, 4, 6, &c. where the first Term is 2, and also the Difference of each Term is 2, I would know what will be the twentieth Term, whose Distance consequently from the first Term is 19; wherefore $19 \times 2 = 38$, and $38 + 2 = 40$, which will constitute the 20th Term required in the Progression.

LEMMA.

The Sum of any Series in Arithmetical Progression may be obtained by adding the greatest Terms together, and multiplying half that Sum into the Number of Terms, or the whole Sum into half the Number of Terms.

PROBLEM XXX.

SUppose One sets out from London, *and travels one Mile the first Day,* 2 *the second,* 3 *the third, &c. And* 5 *Days after another sets out from the same Place, and travels at the Rate of* 12 *Miles a Day. How long, and how far must the first have travelled before he was over taken by the second?*

SOLUTION.

For the Days travelled by the first put x.
by the second $5-x$.
Miles travelled by the first $xx+\frac{x}{2}$.
by the second $12x-60$.

But before this Problem be brought to an Equation, it may not be improper to premise the following Observations.

1. You

1. You must observe that x, the Days travelled by the first, is equal both to the Number of Terms in the Progression, and also to the last Term. Then, *per Lemma*, the Sum of the whole Series is $\frac{x^2+x}{2}$. For 1 is the first Term, and x the last, and their Sum is $x+1$, and $x+1\times\frac{x}{2}=\frac{xx+x}{2}$, the Sum of the whole Series or Number of Miles the first travelled.

2. If the last had travelled the same Number of Days as the first, he had travelled $12x$ Miles; but as he set out 5 Days after, he must travel 5 times 60 Miles less than if he had travelled as many Days as the first; so he travelled $12x-60$. Then considering the Problem, we have this

Equation	1	$\frac{xx+x}{2}=12x-60$.
reduc'd, &c.	2	$xx-23x=-120$.
$2c\square$	3	$x^2-23x+\frac{529}{4}=-120+\frac{529}{4}$.
that is	4	$x^2-23x+\frac{529}{4}=\frac{-480+529}{4}=\frac{49}{4}$.
4ѡ2	5	$x-\frac{23}{2}=\pm\frac{7}{2}$.
$5+\frac{23}{2}$	6	$x=\frac{23\pm7}{2}=15$, or 8.

PROBLEM XXXI.

OUT of a common Pack of Cards (viz. 52.) *let part be distributed into several distinct Heaps, in the manner following: Upon the lowest Card of every Heap, let as many others be laid as are sufficient to make up the Number* 12; *as if* 4 *be the Number of the bottom Card, let* 8 *others be laid upon it; if* 5, *let* 7; *if a, let* 12—*a*, &c. *'Tis required after having given the Number of Heaps, which we call n, as also the Number of Cards left in the Dealer's Hand, which we shall call m, to find the Sum of the Numbers of all the bottom Cards?*

SOLUTION.

Let a, b, c, d, e, f, &c. represent the bottom Cards of the several Heaps; then will $12-a$ be the Number of Cards lying upon a, the first bottom Card; therefore $13-a$ will be the Number of Cards in the first heap. In like manner $13-b$ will be the Number of Cards in the second Heap, $13-c$ in the third Heap, &c. Therefore if n be the Number of heaps, $13n-a-b-c-d-e-f$, &c. will be the Number of Cards in all the Heaps; and if x represents the Sum of all the bottom Cards, $a+b+c+d+e+f$, &c. then $13n-x$ will be the Number of Cards contained in all the Heaps. But the Cards contained in all the Heaps will be the whole Pack, except the Number still remaining in the Dealer's Hands; therefore $52-m$ will also represent all the Cards dealt out into the several Heaps, whence we have this

Equation	1	$13n-x=52-m.$
$1\pm$	2	$x=13n-52+m.$

But $13n$ is $=n\times13$, and $52=4\times13$; Th. $13n-52=\overline{n-4}\times m$; Theref. $x=\overline{n-4}\times13-m$, whence we have the following

THEOREM.

Subtract four from the Number of Heaps, multiply the rest by 13, and to the Product add (m) the Number of Cards remaining in the Dealer's Hands, and you'll have (x) the Number of all the bottom Cards.

EXAMPLE.

Suppose the Number of Heaps 3, and the Number of Cards left in the Dealer's Hands 30; what was the Sum of the Number of all the bottom Cards?

Answer 17. For $13\times-1=-13$, and $30-13=17$.

PROBLEM XXXII.

YOU that are skill'd in mathematic Arts,
Divide ten thousand into two such Parts,

When

When each of them the other hath divided,
Both Quotients make just 5 when right decided?

SOLUTION.

Put $a = 10.000$, $b = 5$, $x =$ one Part; by *Ax.* 3. $a - x =$ other Part.

Equation	1	$\frac{x}{a-x} + \frac{a-x}{x} = b.$
$1 \times \overline{a-x}$	2	$x + \frac{aa - 2ax + xx}{x} = ab - bx,$
$2 \times x$	3	$xx + aa - 2ax + xx = abx - bxx.$
$3 \pm$	4	$2x^2 + bx^2 - 2ax = -aa.$
$4 \div 2 + b$	5	$xx - ax = -\frac{aa}{2+b}.$
$5 c \square$	6	$x^2 - ax + \frac{1}{4}a^2 = \frac{1}{4}a^2 - \frac{a^2}{2+b}.$
$6 \text{ ɯ } 2$	7	$x - \frac{1}{2}a = \pm\sqrt{\frac{1}{4}a^2 - \frac{aa}{2+b}}.$
$7 + \frac{1}{2}a$	8	$x = \frac{1}{2} \pm \sqrt{\frac{1}{4}aa - \frac{aa}{2+b}} = 8273.26$ or $1726{,}74$, the two Numbers.

PROBLEM XXXIII.

ONE lays out 33l. 15s. *in Cloths, which he sells again at the Rate of* 2l. 8s. per *Piece, and gain'd as much by the whole Bargain as a single Piece cost him;* Quere, *how did he buy in his Cloth?*

SOLUTION.

Let 33*l.* 15*s.* = 675 Shillings $= a$, 2*l.* 8*s.* = 48*s.* $= b$,
Number of Shillings given for every Piece x.
His gain *per* Piece $b - x$.

Proportion, $x : b - x :: a : \frac{ab - ax}{x}$, his whole gain, which by the Prob. $= x$. Then this

Equation

Equation	1	$\frac{ab-ax}{x}=x.$
$1\times x$	2	$ab-ax=xx.$
$2+ax$	3	$xx+ax=ab.$
$3c\square$	4	$xx+ax+\frac{1}{4}aa=ab+\frac{1}{4}aa.$
4ɯ2	5	$x+\frac{1}{2}a=\sqrt{ab+\frac{1}{4}aa}.$
$5-\frac{1}{2}a$	6	$x=\sqrt{ab+\frac{1}{4}aa}-\frac{1}{2}a=45.$

He gave 45 Shillings *per* Piece, and ſold them for 48*s.* *per* Piece, and he bought 15 Pieces, therefore he gain'd 45*s.* for 45 : 1 :: 675 : 15,

Some of the foregoing Problems, with many more ſuch, may be reſolved much eaſier by other Methods *peculiar* to each.

But our Deſign is only to acquaint the Reader with *general Ways* of reſolving ſuch *Problems* in a School-Boy's Method, whence he may at pleaſure draw variety of *particular* ones for his Practice. For we have in every Thing ſtudied Brevity with Perſpicuity, ſo that we doubt not but a Learner, will find, with Advantage, that the Whole may ſerve as an uſeful *Introduction* to further mathematical Enquiries, for which alone this was purely wrote.

It may perhaps be expected that we might have inſtructed our Reader in Problems wherein Surds are concerned, and whereby we might have advanced *Converging Series*; but as theſe Subjects are peculiarly handled in a Book entitled *Syntagma Matheſeos*, we ſhall refer our Reader thereto.

It might alſo be expected that we ſhould have ſhown the Application of *Algebra* to *Geometrical Problems*; but as this would ſwell the Book too big, and that Subject is ſo well managed by abler Pens in a quarterly Book entitled *Miſcellanea Curioſa Mathematica*, we have purpoſely omitted them.

FINIS.

BOOKS *printed for, and Sold by* JOHN FULLER, *at the* Bible *and* Dove *in* Ave-Mary-Lane, London.

1. DR *Harris*'s Compleat Collection of Voyages and Travels, 2 Volumes in Folio.

2. *Syntagma Mathesios*: Containing the Resolution of Equations, howsoever complex: With a new Way of solving cubic and biquadratic Equations, analytically and geometrically. Also the universal Method of Converging Series, after an easy and expeditious Manner; in which are treated, the Series for trigonometrical Operations; some new useful Properties in Conic Sections, Center of Oscillation, Percussion, *&c.* Sir *Isaac Newton*'s Method of Fluxions illustrated, direct and inverse Method of the Laws of centripetal Forces, the Investigation of a curious Collection of exponential Equations, and other Problems in the abstruse Parts of the Mathematics; adapted to all those who would learn Algebra, and become Proficients in the most abstruse Branches thereof. By a Gentleman, a Lover of the Mathematics. In 8vo. Price sew'd 5*s*.

3. The Nature and Laws of Chance, wherein the Subject is fully, yet concisely handled; and the more abstruse and important Problems, as well as the more simple ones, are resolved in a general and conspicuous Manner: Containing the Doctrine of Combinations and Permutations clearly deduced; an Investigation of the Probability, that a proposed Event happens a given Number of Times in a given Number of Tryals; a new and very comprehensive Problem of great Use in Lotteries, Cards, *&c.* with others for determining the Probability of Winning, whether at Bowls, Coits, Raffles,

&c.

&c. in any Circumstance of the Play: A Problem for finding the Tryals wherein it may be undertaken, that a proposed Event shall happen or fail a given Number of Times; another to find the Chances for a given Number of Points with a given Number of Dice; and another on the Duration of Play; being three of the most curious and remarkable on the Subject, and all solved by new and general Methods. The Solution of a Problem proposed to the Publick some time ago, in *Latin*, as a very difficult one; with full and clear Investigations of the two new Problems added to the End of Mr *de Moivre*'s last Edition, the Demonstrations of which that Author was pleased to reserve to himself, and whereof he makes such particular Mention. By *T. Simpson*. Pr. 3*s*. stitch'd.

4. Miscellanea Curiosa Mathematica. Number IV. Containing, An Essay on Triangles inscrib'd in, and circumscrib'd about two given Circles. A Calculation of the Lunar Eclipse, that happens *August* 19, 1746. A Solution to Question 131 in the *Ladies Diary*. Solutions to the Questions in Number III. with Tables of Sines, Cosines, &c. also a Solution to an exponential Problem. A Collection of new Questions to be answered in N° V. An easy, familiar and expeditious Method for finding Fluents, with Examples to all Mr *Emerson*'s Forms, designed for Beginners. Two new Measures subservient to mechanical Practice. Price 1*s*.

Number I. Contains, among other useful Matters, a Discourse on the Velocity and Forces in Bodies in Motion; Theorems for determining the Sun's Parallax; An universal spherico-catoptric Theorem; A Theorem on Increments; Of the Maxima and Minima in the celestial Motions; To find the Sun's horizontal Parallax by the Transit of *Venus* over the Sun's Disk in 1761; A Collection of curious Questions, with Answers to the Questions in the present Year's *Diaries*, and the *Gentleman's Magazine*.

Number II. Solutions to all the Questions in Number I. An Essay concerning the Sums of the Powers of an arithmetical Progression; Solutions to the Questions in the

the *Gentleman's Diary*; and twenty new Queſtions to be anſwered, &c.

Number III. Solutions to Queſtions in Number I. and II. and in the *Gentleman's Diary*; alſo new Queſtions propoſed; The Demonſtration of four Theorems in the *Gentleman's Magazine*, by Mr *Facio*; Tables of Chances for any Number of Points thrown on 1, 2, 3, &c. to 12 Dice, incluſively, with the Method of Computation, &c. &c.

5. A Treatiſe of Conic Sections. Dedicated to the Provoſt, Fellows, and Scholars of the College of *Dublin*. By *Robert Steell*. Price 2*s*. 6*d*.

⁂ As this Book has been recommended, by good Judges, for one of the beſt, as well as plaineſt, in its Kind, it is thought a more effectual Service to the young Student in theſe Sciences could not be done, than to oblige him (the firſt Impreſſion being become very ſcarce) with a new, and much cheaper Edition, which is printed in Quarto to accomodate thoſe who ſhall think fit to bind it up with the *Miſcellanea Curioſa Mathematica*.

6. *Thacker*'s firſt and ſecond Volumes Miſcellany, and the Supplements to the *Gentleman's Diaries*.

6. Prayers and Officcs of Devotion for Families, and for particular Perſons for every Day in the Week, for Children, Servants, Priſoners, condemn'd Malefactors, the Sacrament and Sickneſs. By *Benj. Jenks*, late Rector of *Harley*; the 11th Edition. Price 3*s*. 6*d*.

7. The Leſſer Whole Duty of Man; Or, Chriſtian's Guide to Devotion and Obedience; With Short Prayers for Families and private Perſons who have not Time for longer; alſo a true and proper Preparation for the Sacrament. Printed for the Uſe of the Soldiery, and the poor Inhabitants of *Georgia* and the other Plantations. The Third Edition. Price 4*d*. ſtitch'd, 6*d*. bound, or 14 to the Dozen.

☞ Be careful to obſerve that this *Leſſer Whole Duty of Man* is very comprehenſive, and being conciſe is the more likely to be read thro', and its Contents regarded and remembred by working People; the Price alſo being at moſt but 6*d*. twenty Shillings laid out in this Book

by the Rich and Well-disposed will do more good, as it will go ten times farther than in the five Shilling Duty of Man, and eight time farther than the four Shilling one, both which upon a nice Calculation will be found so many Degrees *heavier*, (*to carry*) and will, it's fear'd, be thought too tedious in this present *Wicsted* Age, when a great tho' good Book is reckoned a great Drug, unless well puffed by the *King's Authority*.

8. The Shepherd's Kalender, containing twelve Eclogues, proportionably to the twelve Months; and a Cut suitable to each Month. By *J. Spencer*. Price 5*s*.

9. Eutropii Historiæ Romanæ Breviarium, ab Urbe Condita usque ad Valentinianum, & Valentem Augustos; cum Notis. Editio decima. Price 1*s*.

10. *English* Exercises for School-Boys to translate into *Latin*, comprizing all the Rules of Grammar, and other necessary Observations, ascending gradually from the meanest to higher Capacities. By *J. Garretson*. The 19th Edition. Price 1*s*.

11. The Church Catechism, with single Proofs, Graces and Prayers, by Mr *Bunbury*, 12mo. Price 2*d*. or 12*s*. *per* Hundred, to those that give them away.

13. The Companion to the Altar, to bind up with Common Prayers of all Sizes.

14. The *English* School Reform'd: In six Parts. The 12th Edition. By *R. Brown*, in 8vo.

15. An Alarm to the unconverted Sinners. By *J. Alleine*. In 12mo.

16. Nepos Delphini. In 8vo.

Likewise may be had all Sorts of Bibles, Common-Prayers of different Sizes for Churches and Families, with Cuts or without, neatly bound in all kinds of Binding. Testaments, Psalters, Primers, &c. all Sorts of modern Books in Divinity, Law, Physic, Classics, Poetry, Mathematics, History, in *English* or other Languages, and School-Books; where any Chapman may be supplied at reasonable Rates by Wholesale and Retail.

Note, Chapman's Books, *Burton*'s Books, Wholesale or Retail: Where is kept the Paste-Board Warehouse.

A-Be. 55-4
13.26
1 3.0.

a Carn
Jdh
Carle

29.628
27|190
189
343|1291
29

182
182
364
1456
192
331 24
4096
290.28